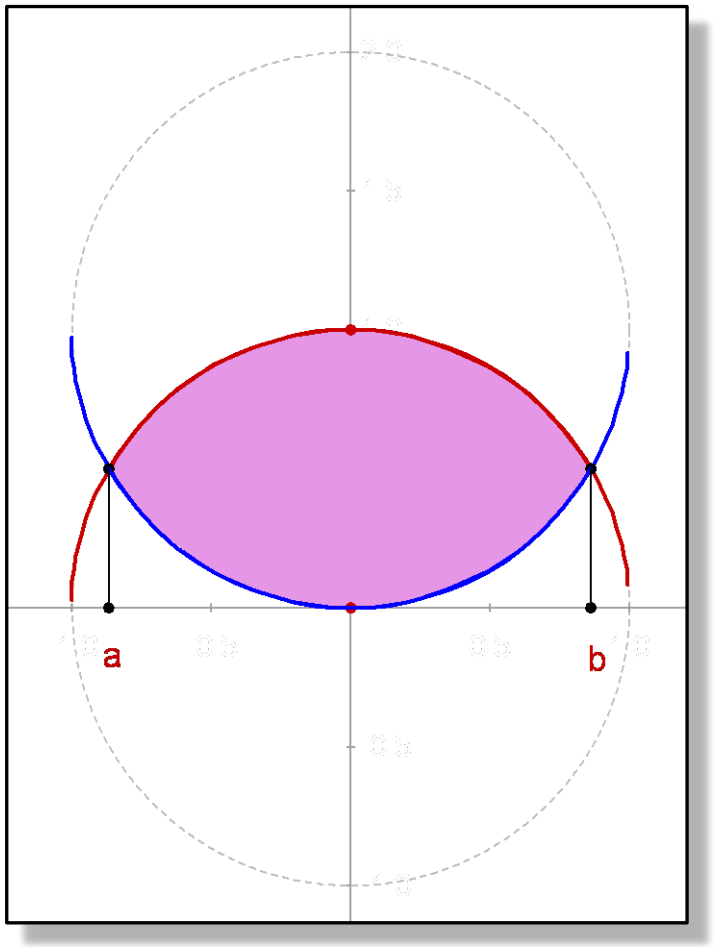
Learning Calculus With Geometry Expressions™

by L. Van Warren



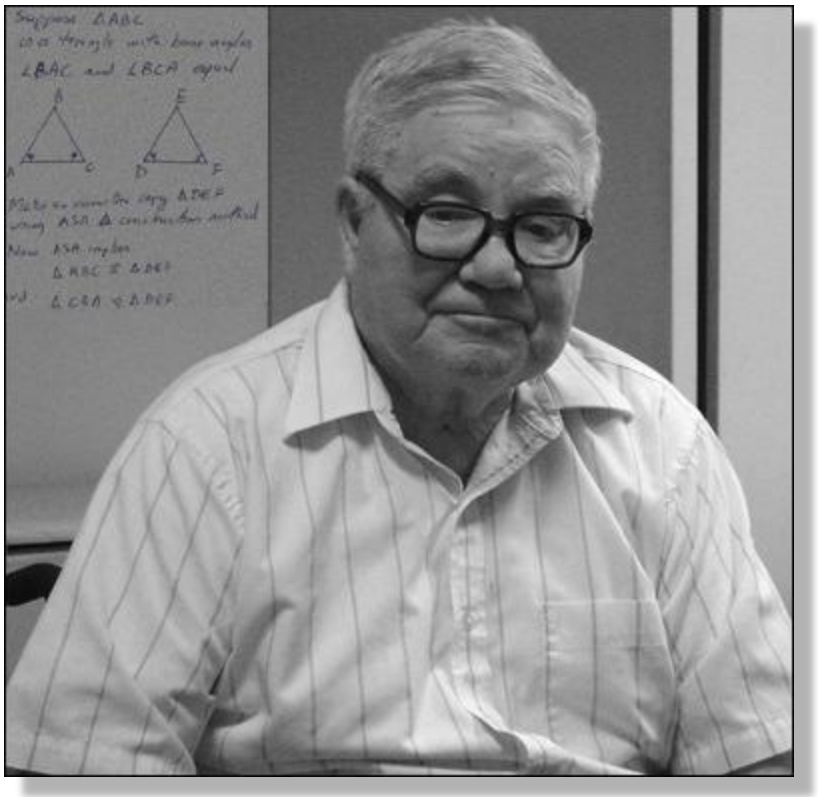
Lecture 26:
The Area Between Curves

Chapter 6: Integration Techniques

Lecture	Topic
23	Integration: Cartesian and Polar
24	Integration by Substitution
25	Integration by Parts
26	**Area Between Curves**

My Calculus Inspiration

Reo Flaherty
Hall High School Math Teacher

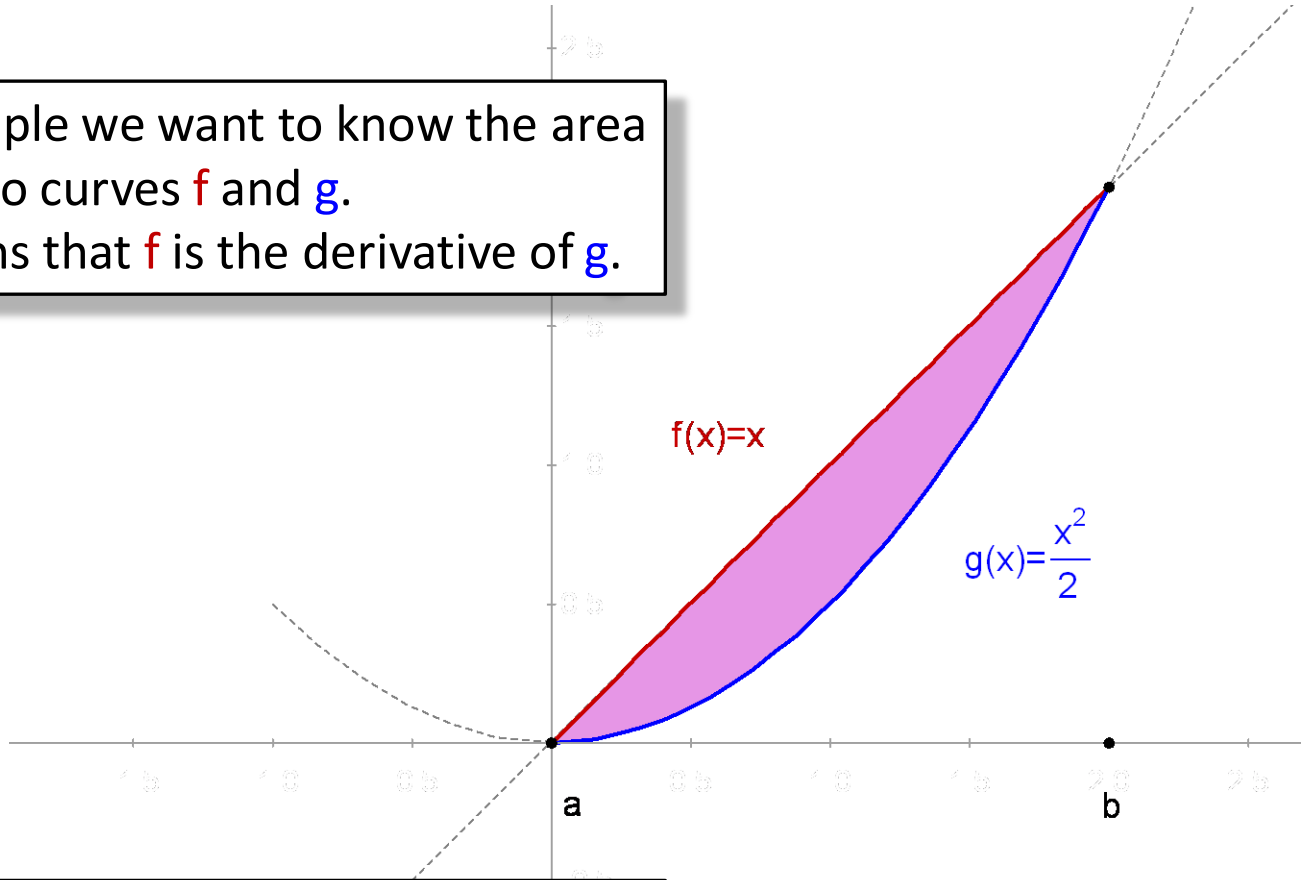


- Excelled at the Constructive Proof and Geometry Thereom Exposition
- Taught 41 Years
- Overcame Charcot-Marie-Tooth which prevented him from grasping chalk in either hand.
- Worked on an Overhead Projector Using Felt-Tipped Pens - Used color to convey meaning in constructions and symbols.

Area Between Curves 101:

Lecture26-AreaBetweenCurves101.gx

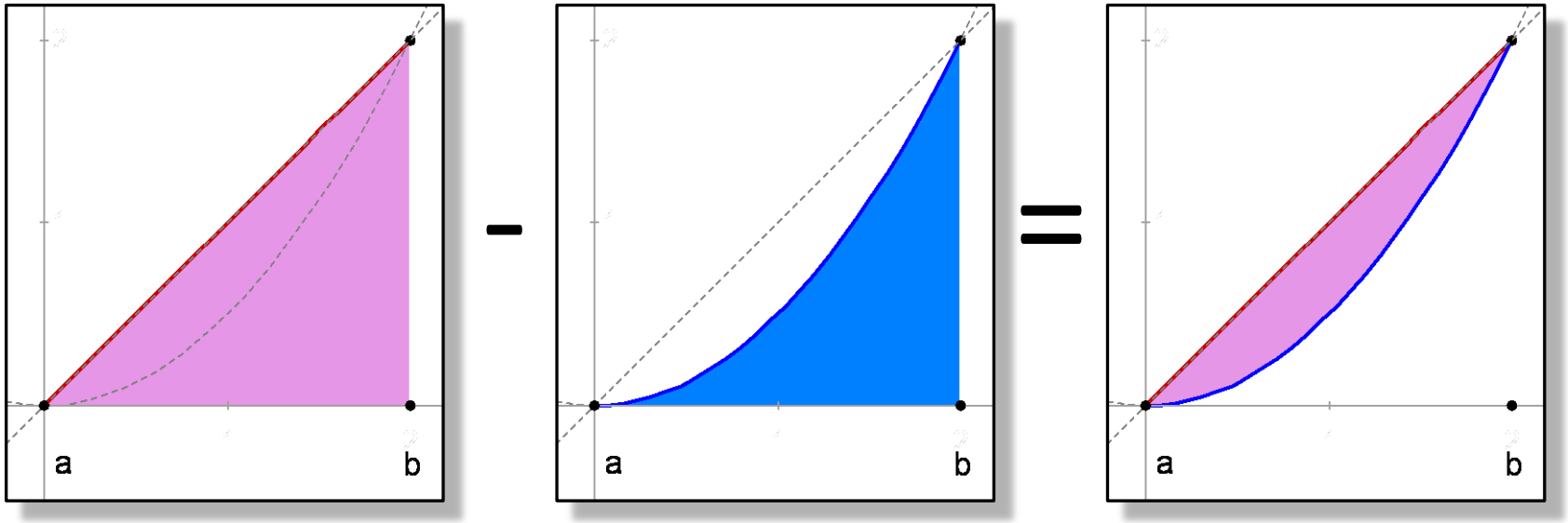
In this example we want to know the area between two curves f and g.
It so happens that f is the derivative of g.



Exercises:
1) Compute the intersection points of f(x) and g(x).
2) Check your work with View→Show All

Area Between Curves 101:

Lecture26-AreaBetweenCurves101.gx

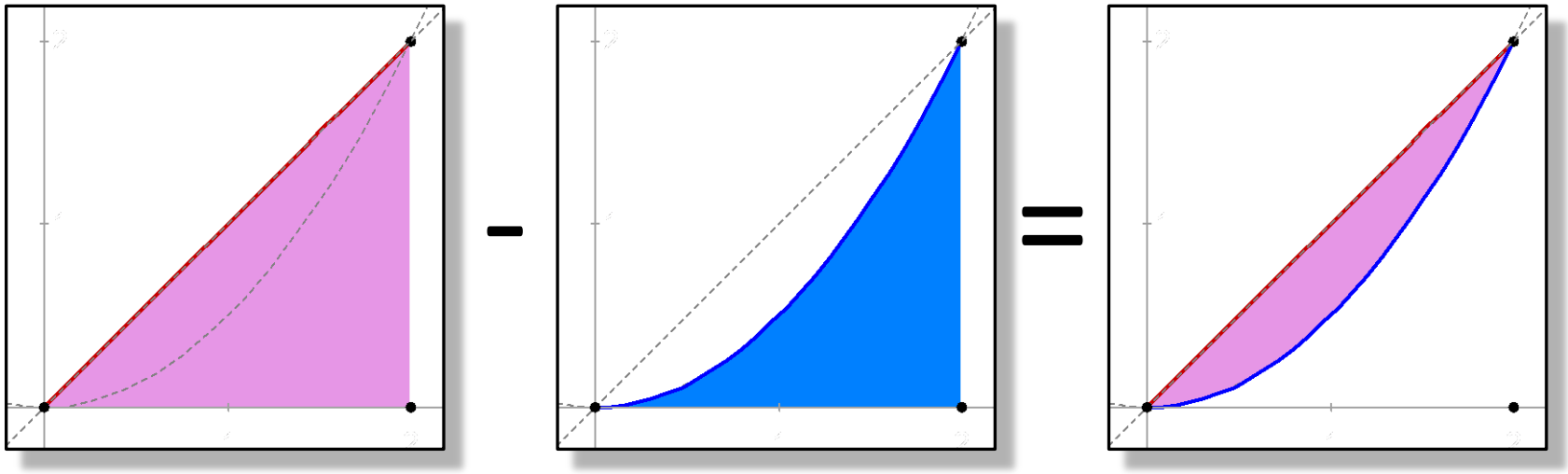


$$\int_a^b f\,dx - \int_a^b g\,dx = \int_a^b (f - g)\,dx$$

Integration is a Linear Operator, thus
The Difference of the Integrals Is The Integral of the Difference

Area Between Curves 101:

Lecture26-AreaBetweenCurves101.wxm



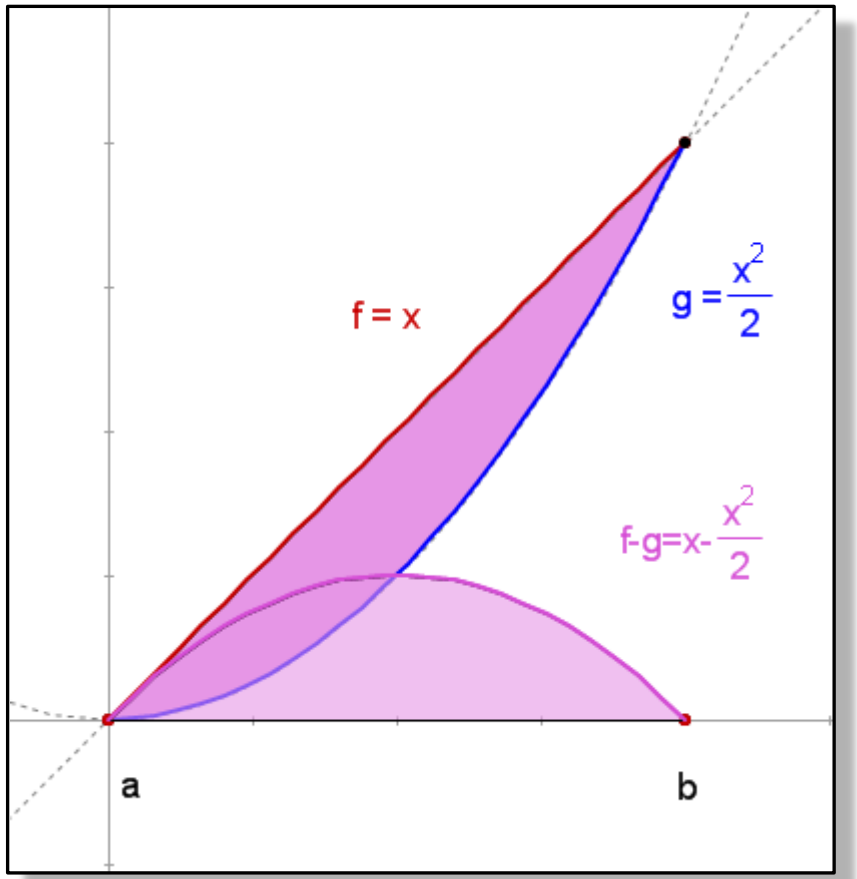
$$\int_0^2 x\,dx - \int_0^2 \frac{x^2}{2}\,dx = 2 - \frac{4}{3} = \frac{2}{3}$$

Exercise:
1) Check this result integrating by hand.
2) Check your work using Lecture26-AreaBetweenCurves101.wxm

Area Between Curves 101:

Lecture26-AreaBetweenCurves101a.gx

The two purple regions have the same area.
One represents a transformation of the other!



Exercises:

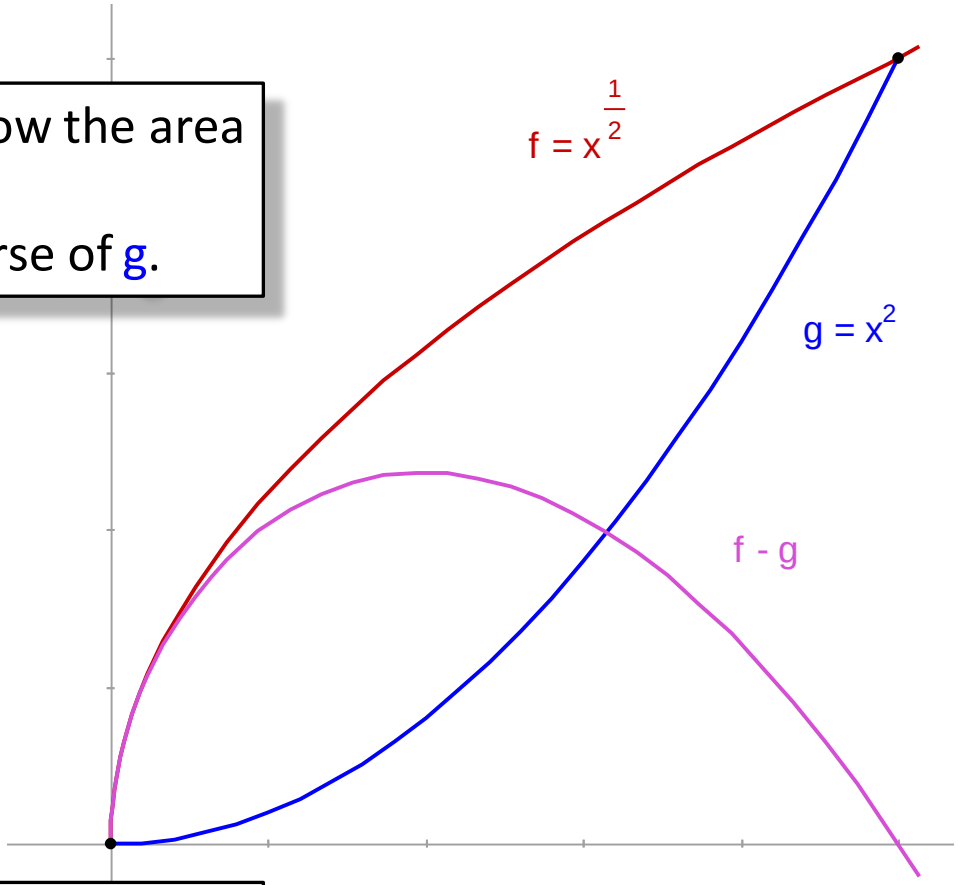
1) Distinguish between the boundary curves and the filled regions by writing the equations for each.

2) Describe how one might rotate, invert and squish the top purple area to produce the bottom purple area.

Area Between Curves 201:

Lecture26-AreaBetweenCurves201.gx

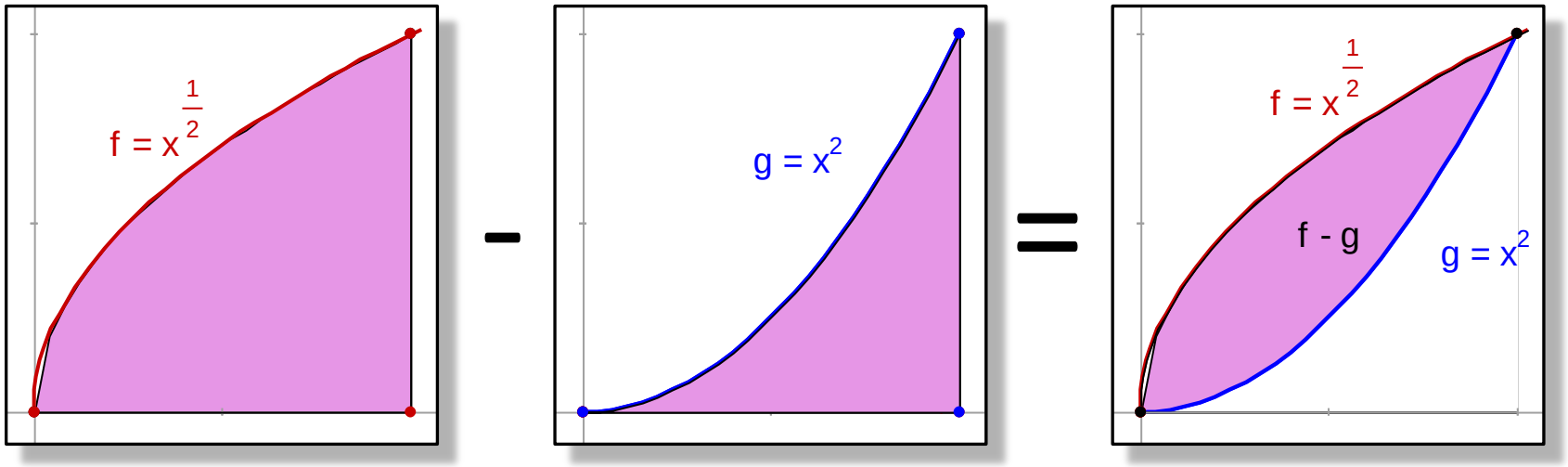
In this example we want to know the area between two curves f and g.
It so happens that f is the inverse of g.



Exercises:
1) Compute the intersection points of f(x) and g(x).
2) Check your work with View→Show All

Area Between Curves 201:

Lecture26-AreaBetweenCurves201.wxm



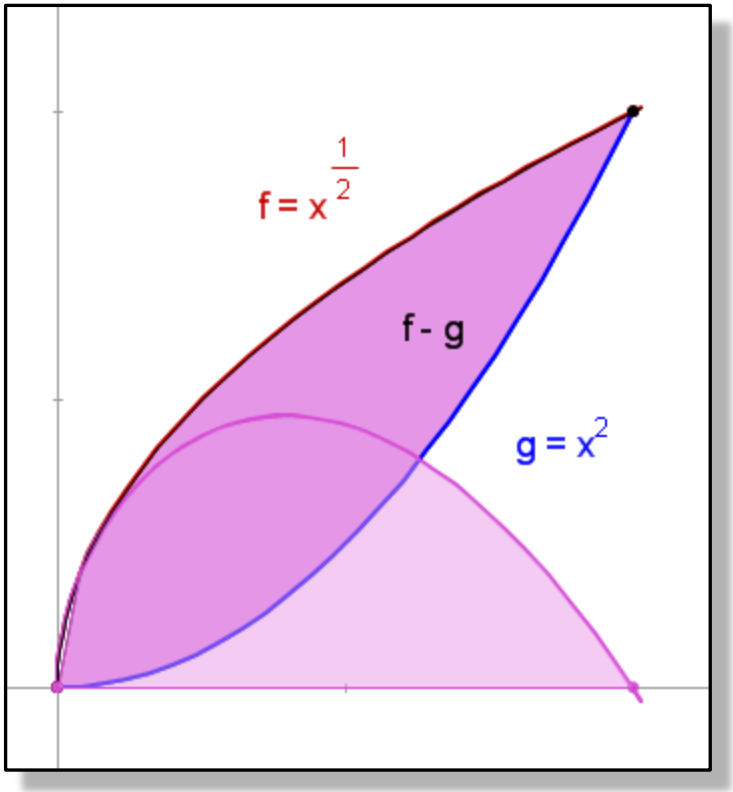
$$\int_0^1 \left(x^{1/2} - x^2\right) dx = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

Exercise:
1) Check this result integrating by hand.
2) Check your work using Lecture26-AreaBetweenCurves201.wxm

Area Between Curves 201:

Lecture26-AreaBetweenCurves201a.gx

Again the two purple regions have the same area.



Exercises:

1) Distinguish between the boundary curves and the filled regions by writing the equations for each.

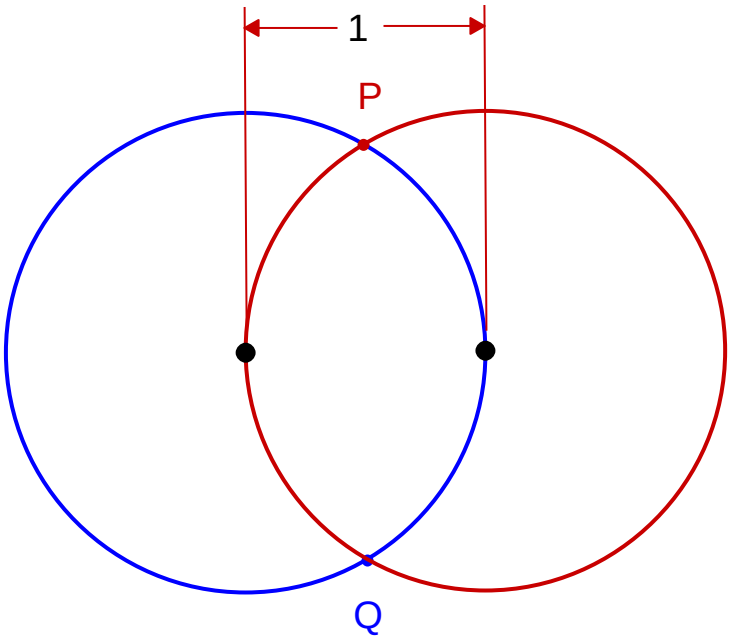
2) Create a symbolic and graphical example where:
$f = a\,x^{1/2}$ and
$g = a\,x^{2}$

3) Compute all intersection points for positive values of the constant a.

Area Between Curves 301:

Lecture26-AreaBetweenCurves301.gx

In this example we want to know the area shared between two circles.
This is the classic AND operator in Boolean algebra.



Exercises:
1) Write the equation of each circle.
2) Compute the intersection points P and Q.

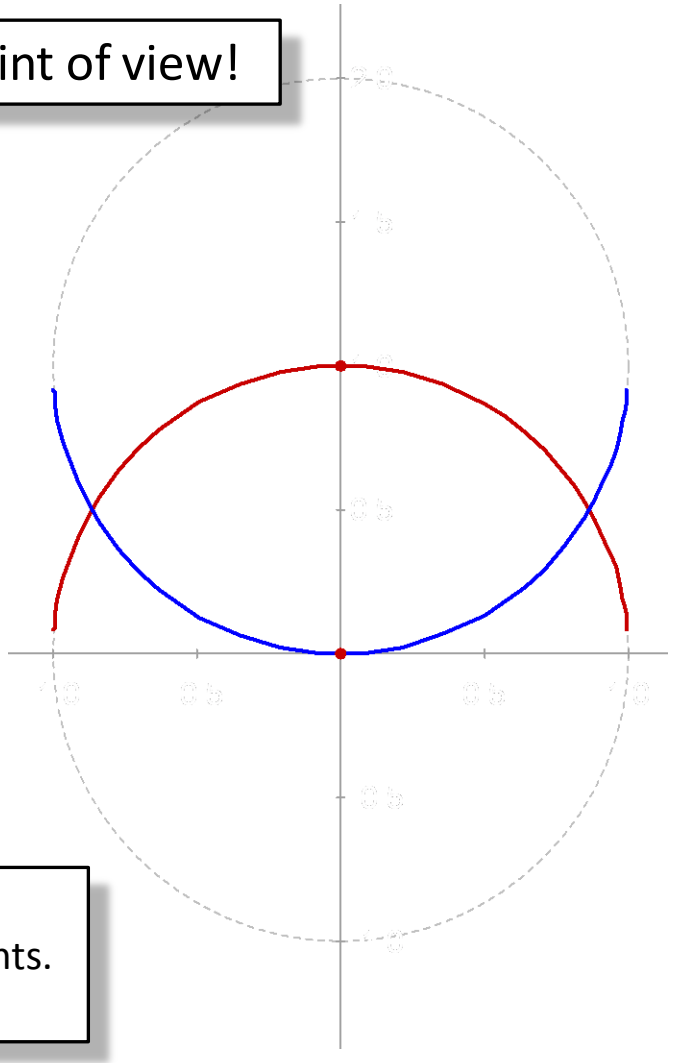
Area Between Curves 301:

Lecture26-AreaBetweenCurves301a.gx

The trick is to carefully choose one's point of view!

$$f(x) = \sqrt{1 - x^2}$$

$$g(x) = -\sqrt{1 - x^2} + 1$$



Exercises:
1) Compute the x values of the intersection points.
2) Check your work using Maxima™.

Using Maxima™:

Lecture26-AreaBetweenCurves301.wxm
Lecture26-AreaBetweenCurves301b.gx

```
f(x):=sqrt(1-x^2);
```

$$f(x):=\sqrt{1-x^2}$$

```
g(x):=-f(x)+1;
```

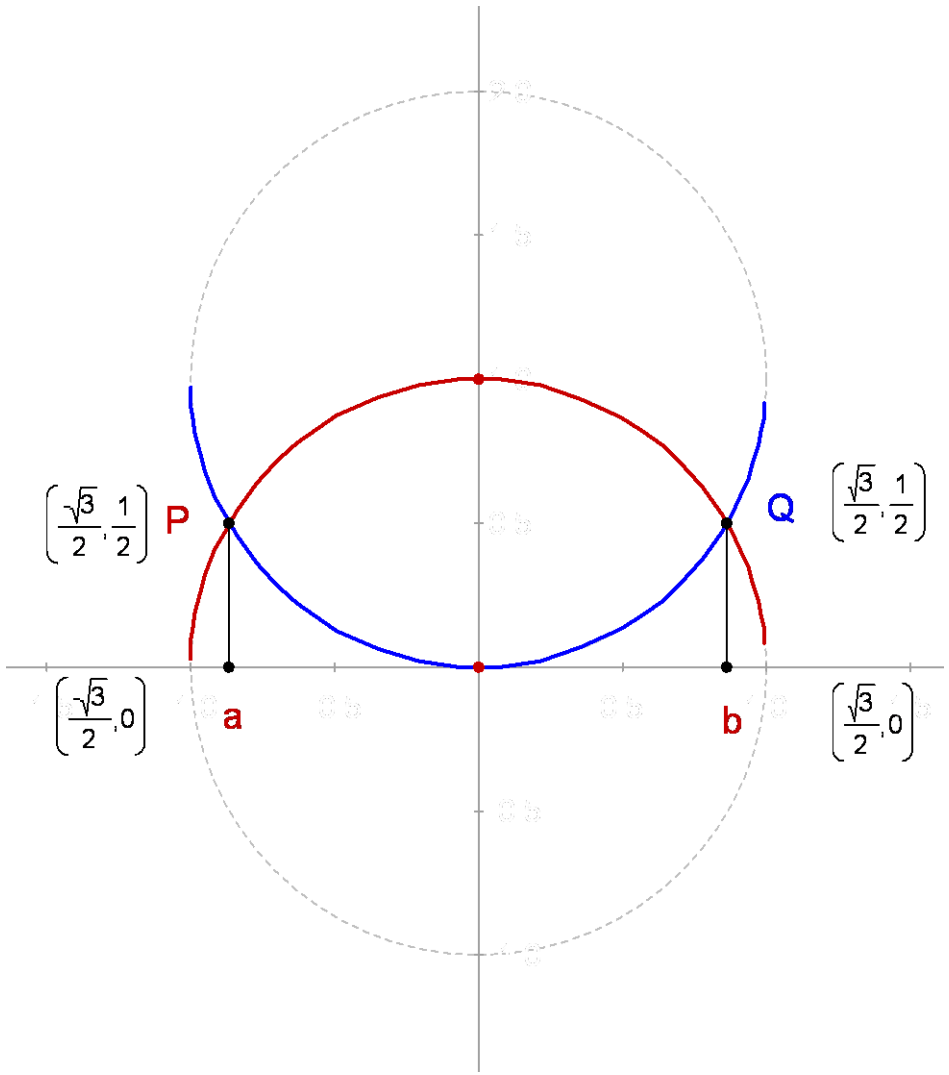
$$g(x):=-f(x)+1$$

```
g(x);
```

$$1-\sqrt{1-x^2}$$

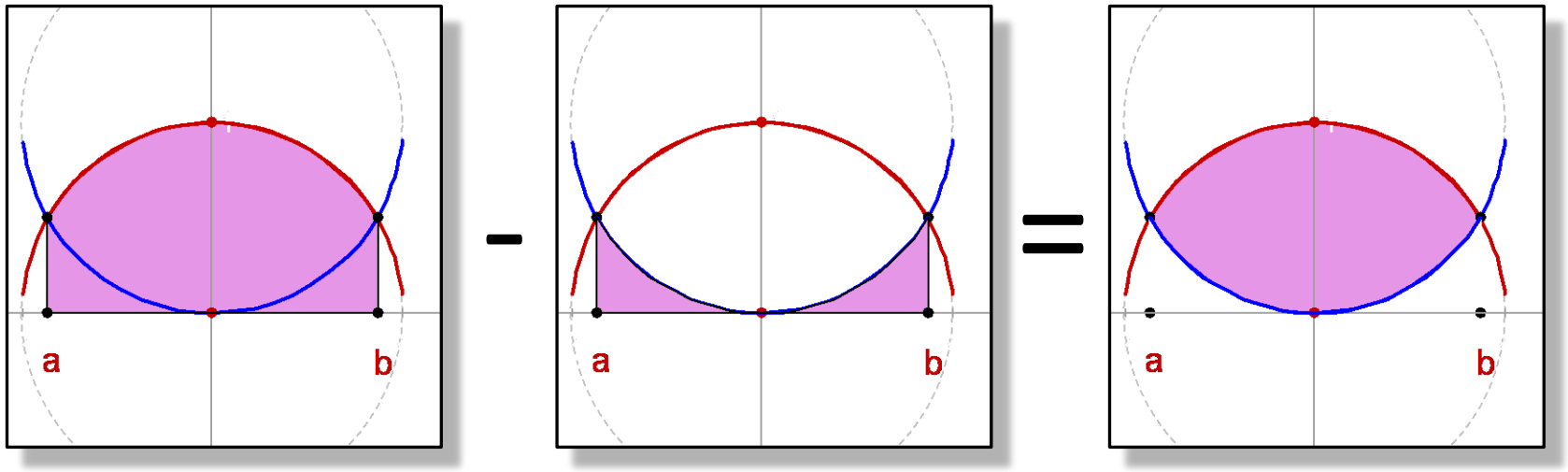
```
solve(f(x)-g(x),x);
```

$$\left[x=-\frac{\sqrt{3}}{2},\ x=\frac{\sqrt{3}}{2}\right]$$



Area Between Curves 301:

Lecture26-AreaBetweenCurves301c.gx
Lecture26-AreaBetweenCurves301a.wxm



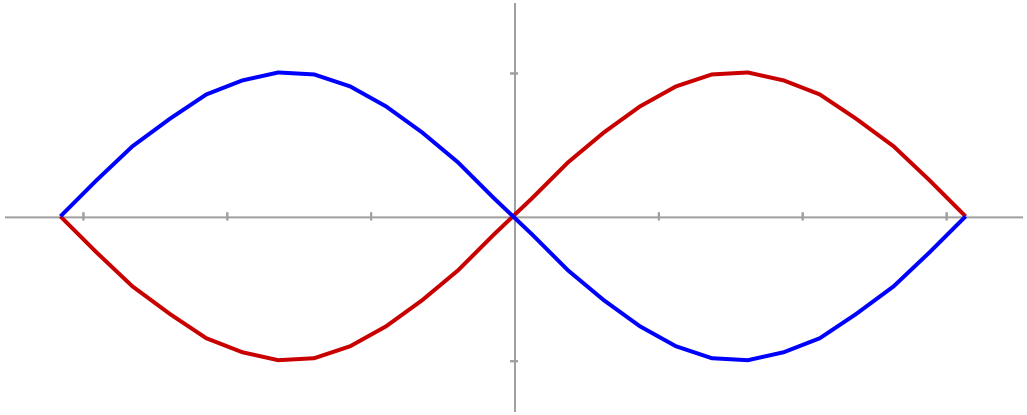
$$\int_a^b (f - g)\,dx = ?$$

Exercise:
1) Compute the result by hand, then check with Maxima™.
2) Draw the difference function f – g.
3) Is the difference function parabolic?

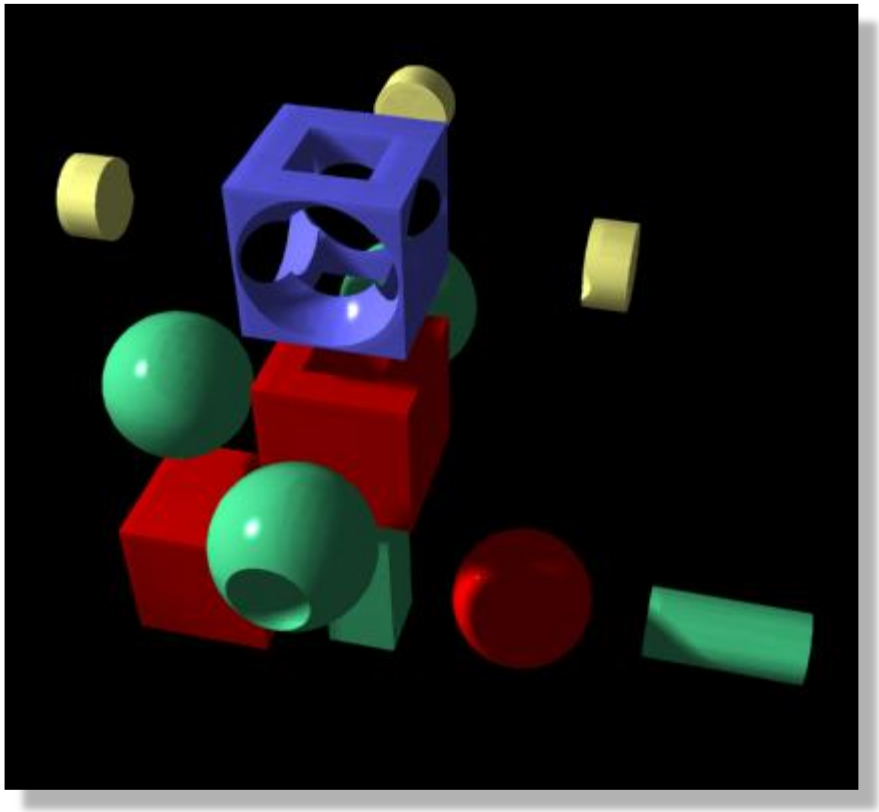
Last Problem:

Lecture26-LastProblem.gx

Without using any tools, compute the area between these two curves:



$$\int_{-\pi}^{\pi} (\sin(x) + \sin(x))\,dx = ?$$



End