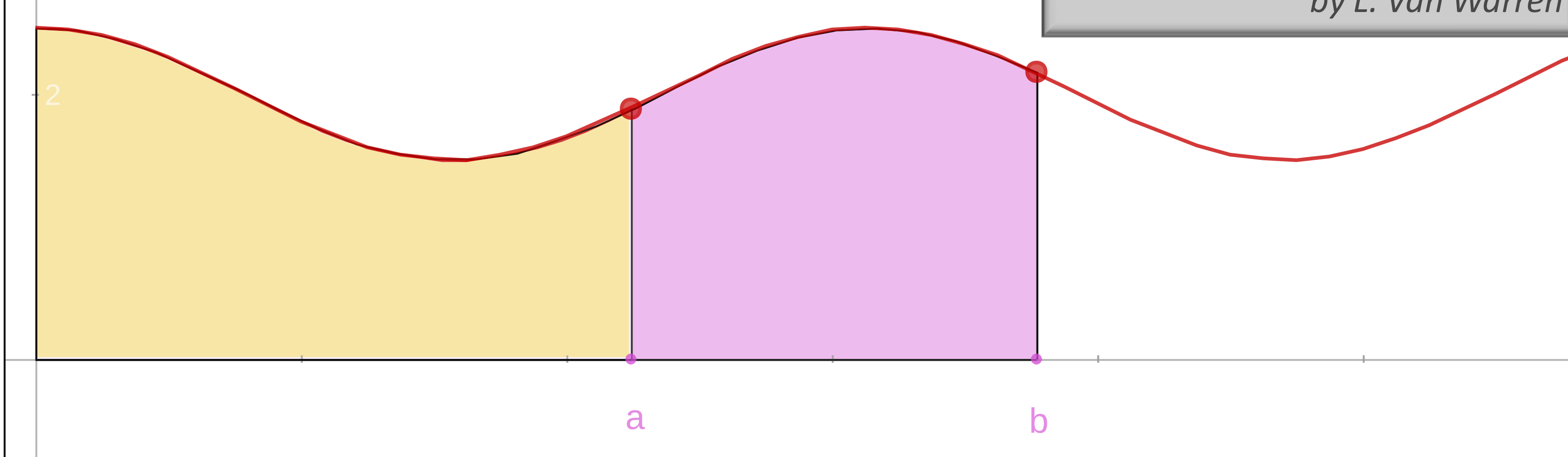
Learning Calculus With Geometry Expressions™

by L. Van Warren

Lecture 22:
The Fundamental Theorem of Calculus

$$\int_a^b f(x)\,dx = F(b) - F(a)$$



Chapter 5: Integration

Lecture	Topic
19	*Antiderivatives*
20	*Integration: Area and Distance*
21	*The Definite Integral*
22	***Fundamental Theorem of Calculus***

My Calculus Inspiration

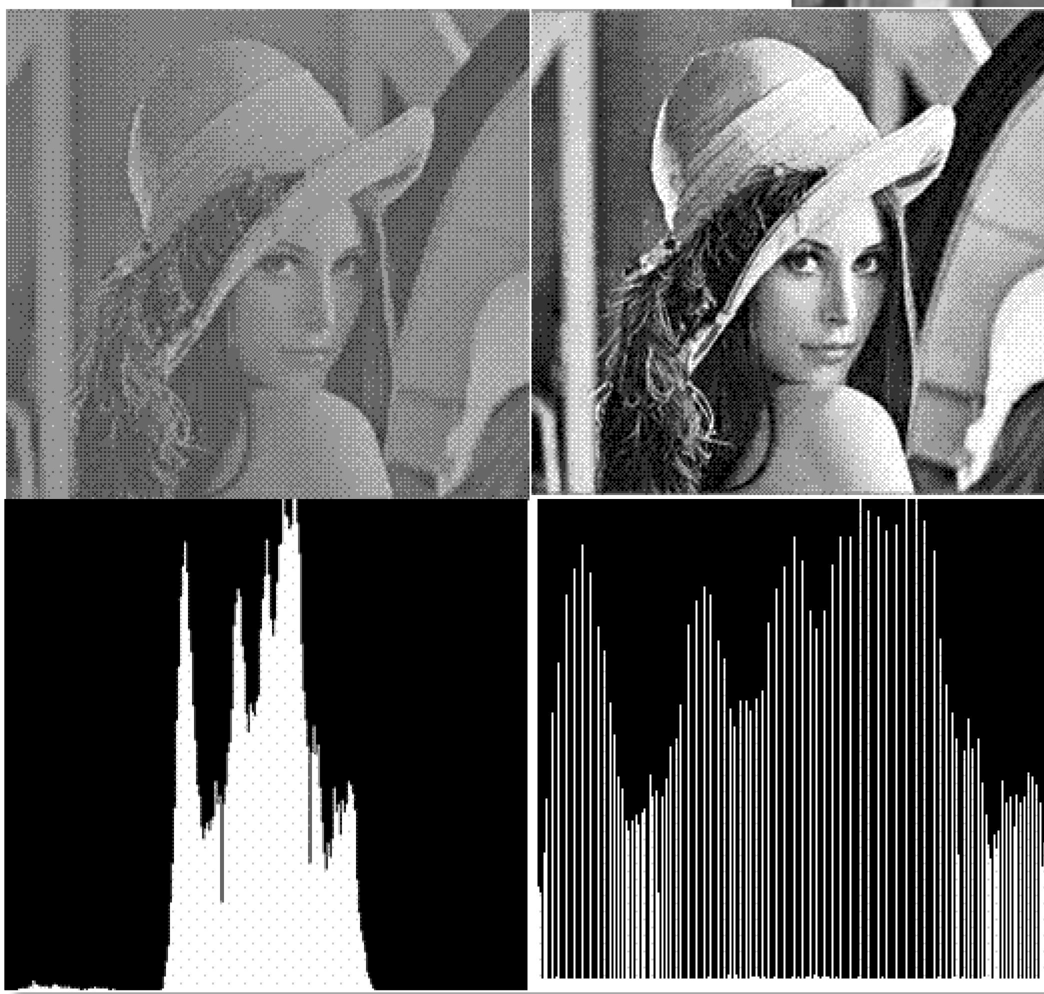
Professor Thomas Stockham

PhD. - MIT 1959
Univ. of Utah. – Salt Lake City

- Father of Digital Audio
- 1974 Investigated Nixon Whitehouse Tapes
- First Commercial Digital Recording
- Emmy 1988
- Grammy 1994

Thomas Stockham

Digital Signal Processing Pioneer



Fundamental Theorem of Calculus:

Lecture22-DerivativeOfIntegral.gx

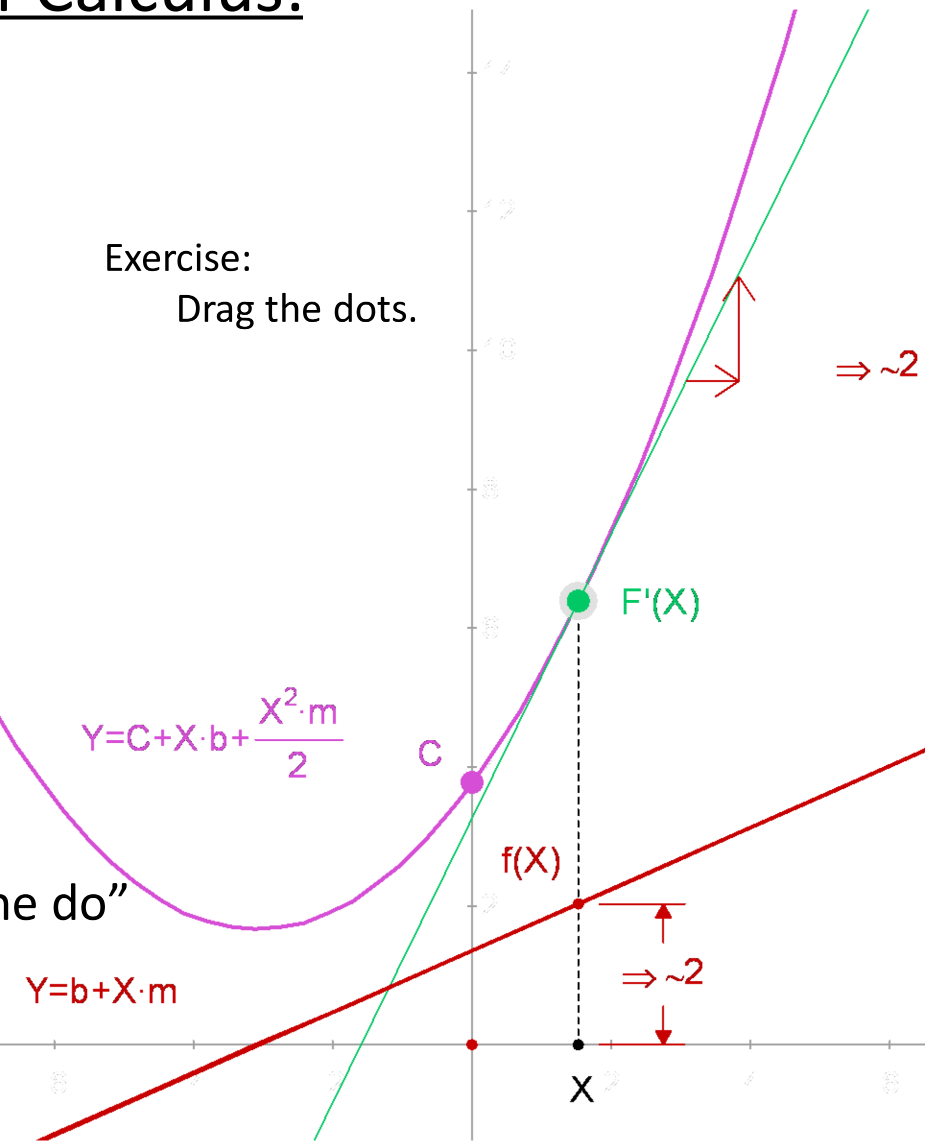
The Rate of Change of
the Area Under a Curve is
the Curve Itself.

OR

The Derivative of
the Integral is
the Function.

Think inverses: When you “undo the do”
you are back where you started.

Exercise:
Drag the dots.



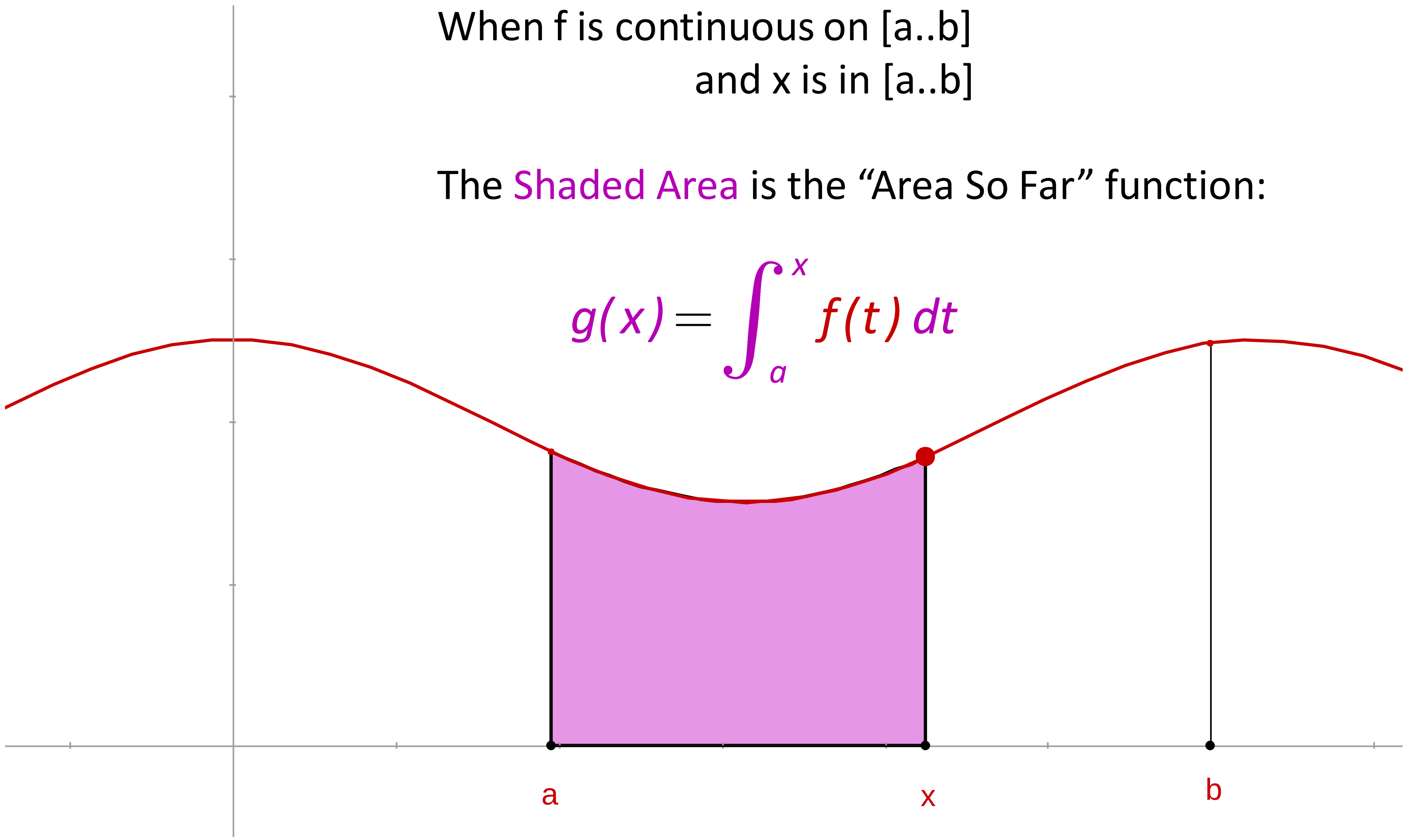
Fundamental Theorem - Part One:

Lecture22-FundamentalTheoremPart1.gx

When f is continuous on [a..b]
and x is in [a..b]

The Shaded Area is the "Area So Far" function:

$$g(x) = \int_a^x f(t)\,dt$$

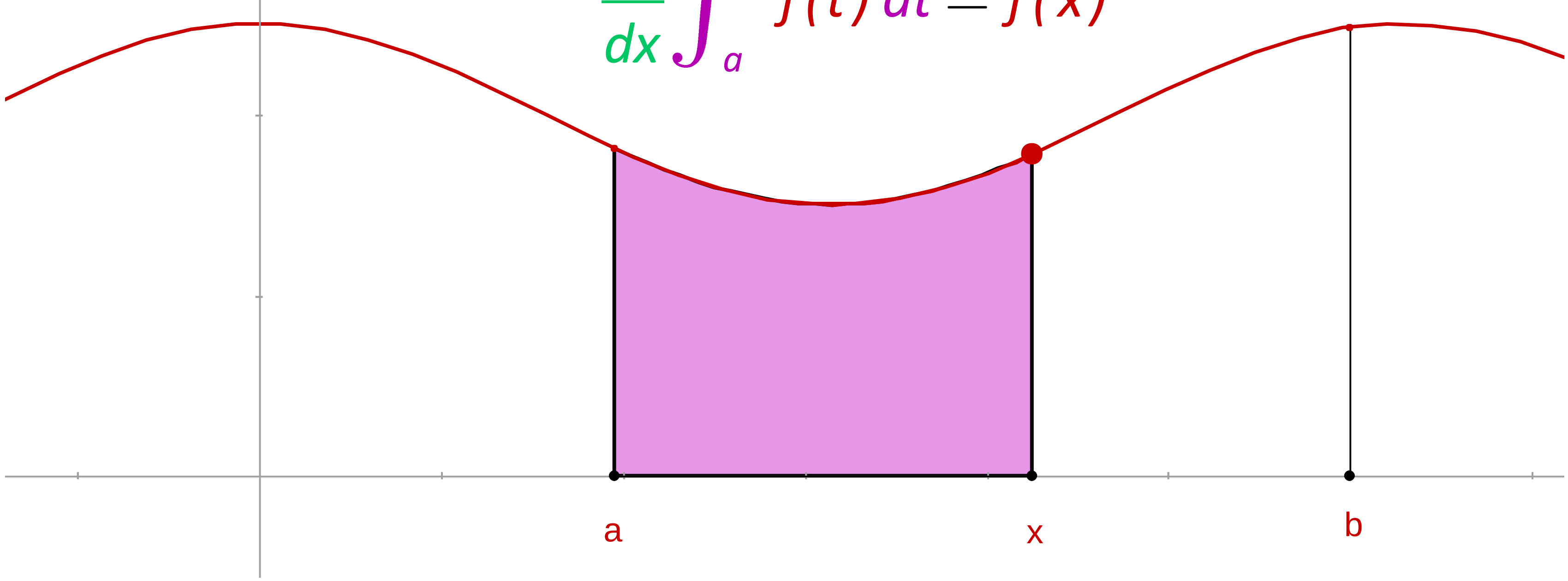


FTC -Part One: Alternate Form

Lecture22-FundamentalTheoremPart1.gx

FTC can be written in the alternate form:

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$



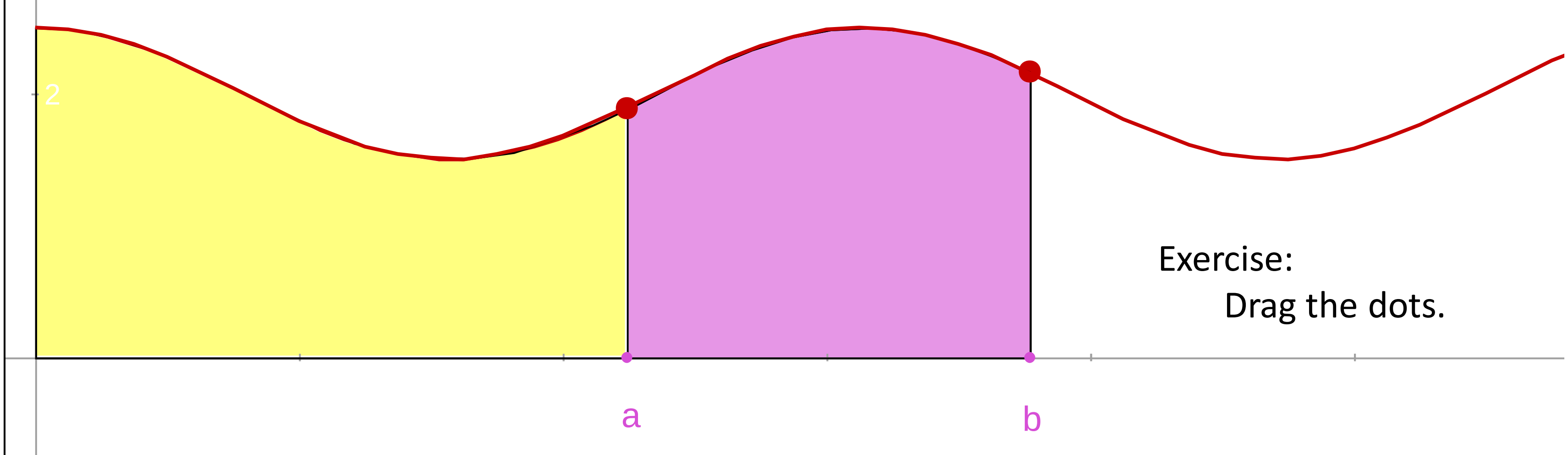
Fundamental Theorem - Part Two:

Lecture22-FundamentalTheoremPart2.gx

Integrating f(x) in the interval from a to b:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$\text{where } F(x) = \int f(x)\,dx$$



Exercise:
Drag the dots.

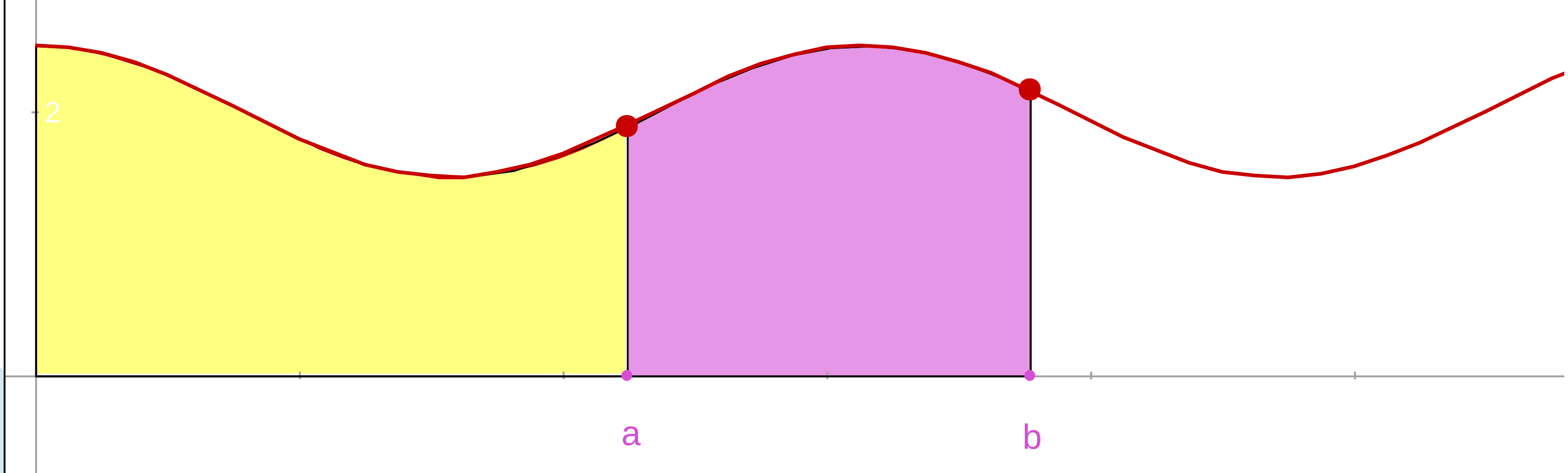
FTC- Part Two: Alternate Form

Lecture22-FundamentalTheoremPart2.gx

FTC part two can also be written:

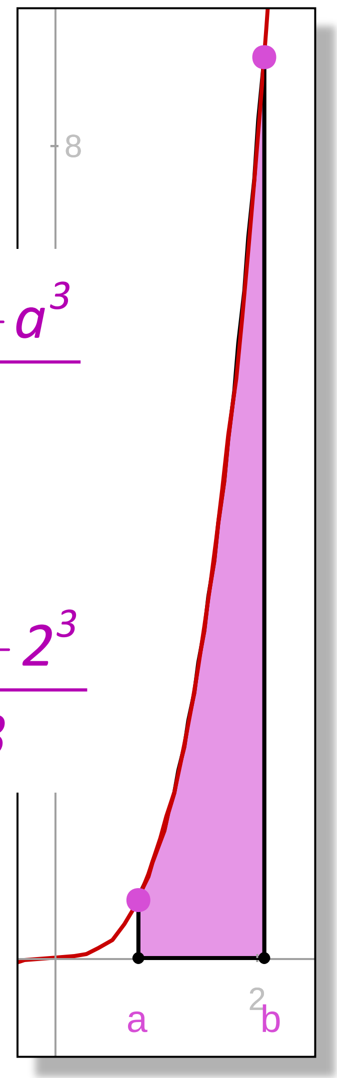
$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

$$\textit{where } F'(x) = \frac{d}{dx}\int f(x)\,dx$$



Evaluating Limits:

Lecture22-EvaluatingLimits.gx



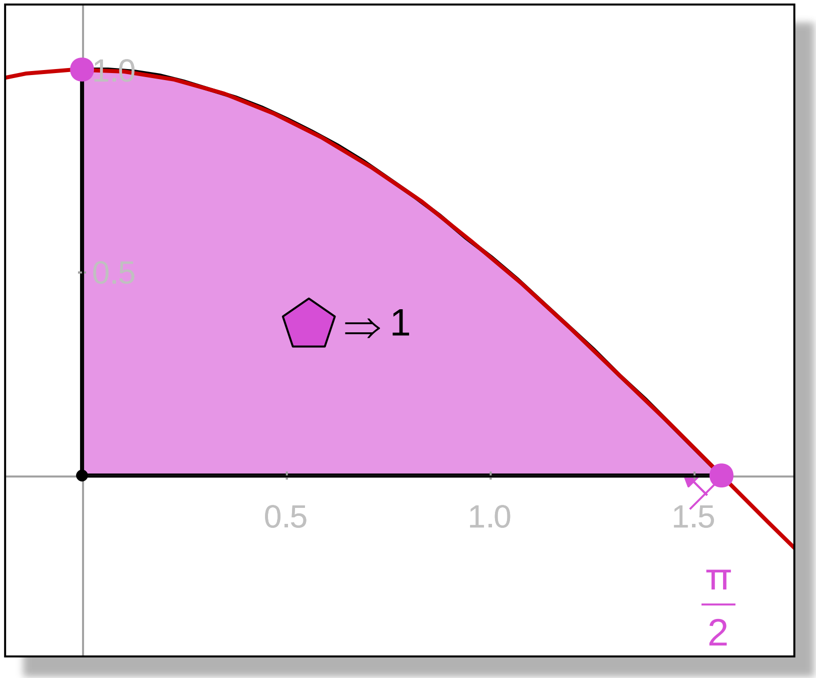
$$\int_a^b x^2\,dx = \left.\frac{x^3}{3}\right|_a^b = \left.\frac{x^3}{3}\right|^b - \left.\frac{x^3}{3}\right|_a = \frac{b^3}{3} - \frac{a^3}{3} = \frac{b^3 - a^3}{3}$$

$$\int_2^4 x^2\,dx = \left.\frac{x^3}{3}\right|_2^4 = \left.\frac{x^3}{3}\right|^4 - \left.\frac{x^3}{3}\right|_2 = \frac{4^3}{3} - \frac{2^3}{3} = \frac{4^3 - 2^3}{3}$$

Evaluating Limits Again:

Lecture22-EvaluatingLimitsAgain.gx

$$\int_0^b \cos(x)\,dx = \sin(x)\Big|_0^b = \sin(b)$$



Exercise:

1) Repeat for sin(x) using Maxima™
2) Create corresponding diagram in Geometry Expressions™.

$$\int_0^{\pi/2} \cos(x)\,dx = \sin(x)\Big|_0^{\pi/2} = 1$$

More Summation Identities

Lecture22-MoreSummationIdentities.wxm

$$\sum_{i=1}^{n} c = c \cdot n$$

$$\sum_{i=1}^{n} c \cdot a_i = c \sum_{i=1}^{n} a_i$$

$$\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$

$$\sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$$

Exercise:
Using these forms, deduce a similar set of identities for integrals.

Definite Integral Identities – Linear Operators

Lecture22-DefiniteIntegralIdentities.wxm

$$\int_a^b c\,dx = c\cdot(b-a)$$

$$\int_a^b c\,f(x)\,dx = c\int_a^b f(x)\,dx$$

$$\int_a^b [f(x)+g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b [f(x)-g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

Exercise:

Execute each case using $a = 1$, $b = \pi$, $c = 2$, $f(x) = \sin(x)^2$ and $g(x) = \cos(x)^2$.

Definite Integral – Limit Identities:

Lecture22-LimitIdentities.wxm

LIMIT INVERSION

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

IDENTICAL LIMITS

$$\lim_{a\to b}\int_a^b f(x)\,dx = 0$$

$$\lim_{b\to a}\int_a^b f(x)\,dx = 0$$

Exercise:

Examine each case using $f(x) = x^2$ in Maxima™.

Definite Integral – Comparison Properties:

IF $f(x) \geq 0$ *for x in [a..b]*

THEN

$$\int_a^b f(x)\,dx \geq 0$$

Exercise:
Create an example
for each of these cases
in Geometry Expressions™.

IF $f(x) \geq g(x)$ *for x in [a..b]*

THEN

$$\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$$

Comparison Properties Cont'd:

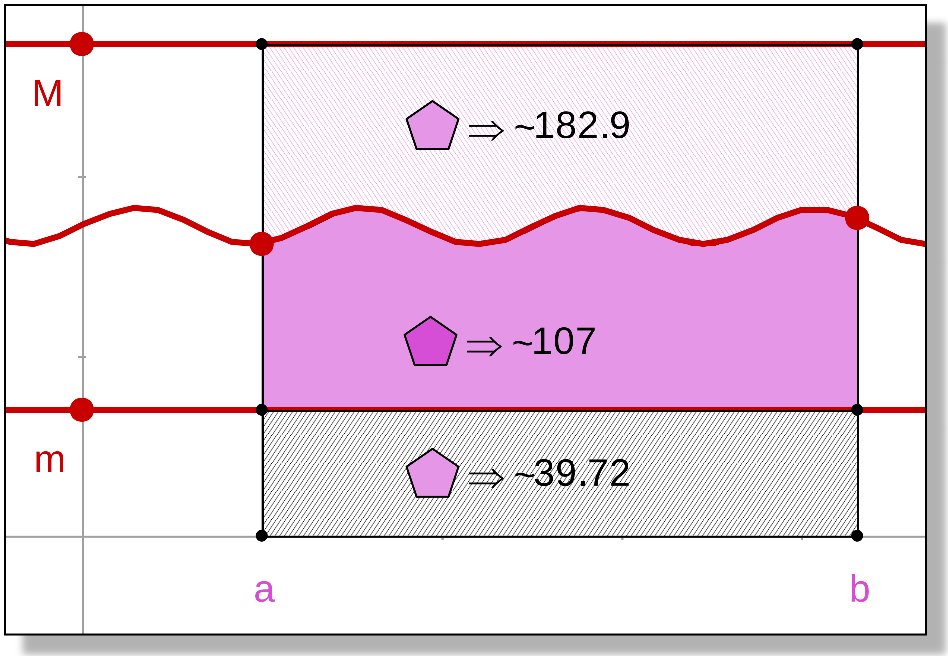
Lecture22-ComparisonProperties.gx

IF $m \le f(x) \le M$ *for x in [a..b]*

THEN

$$m\cdot(b-a) \le \int_a^b f(x)\,dx \le M\cdot(b-a)$$

Exercise:
Drag the dots.



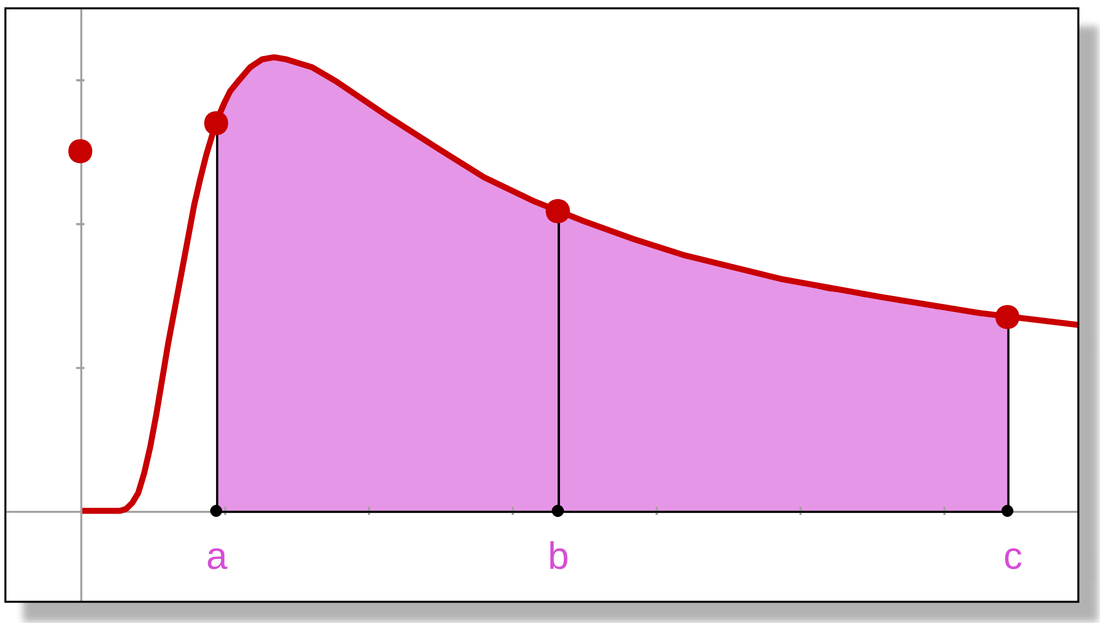
Contiguous Intervals:

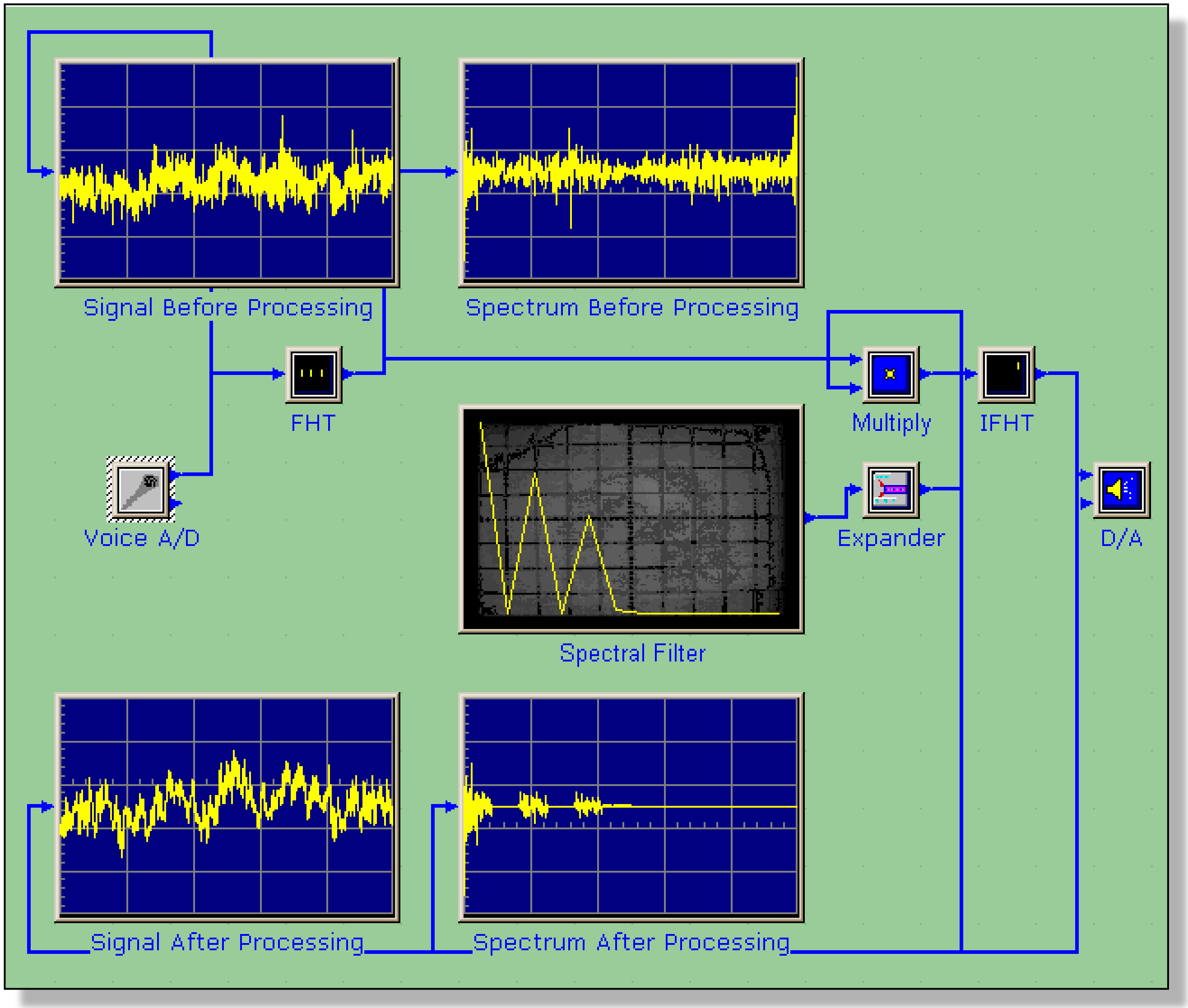
Lecture22-ContiguousIntervals.gx

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

$$\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$$

Exercise:
Drag the dots.




Signal Before Processing
Spectrum Before Processing
FHT
Multiply
IFHT
Voice A/D
Spectral Filter
Expander
D/A
Signal After Processing
Spectrum After Processing

End