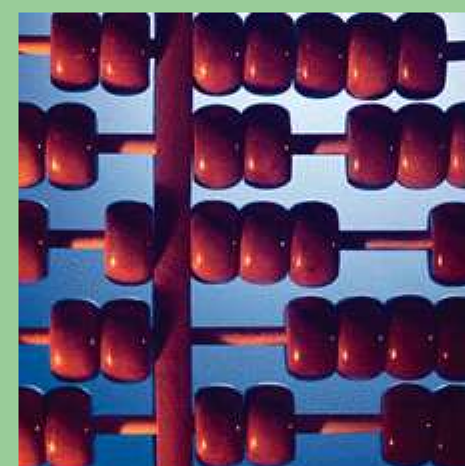
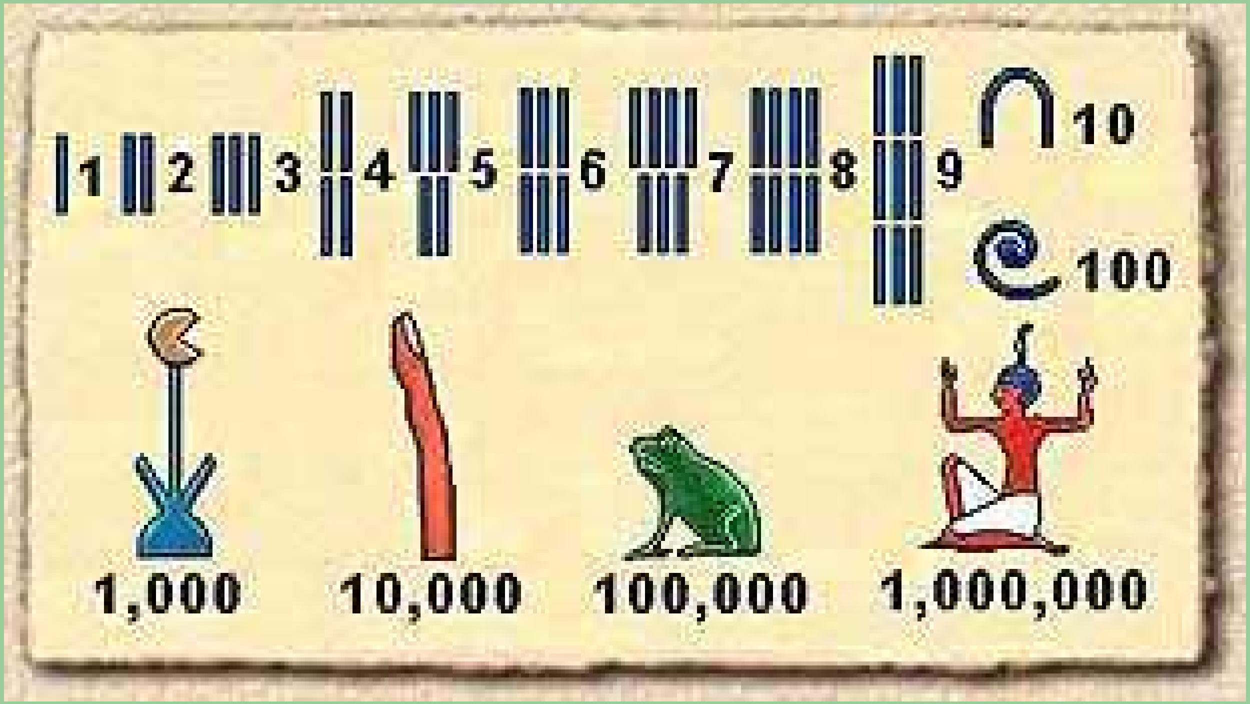
Base One



L. Van Warren

Start at the beginning…



What is a placeholder?

128

$$1 \cdot 100 + 2 \cdot 10 + 8 \cdot 1$$

$$1 \bullet 10^2 + 2 \bullet 10^1 + 8 \bullet 10^0$$

$$1 \cdot 100 + 2 \cdot 10 + 8 \cdot 1$$

$100 \quad + 2 \cdot 10 + 8 \cdot 1$

100 + 20 + 8 • 1

100 + 20 + 8

100 20 8

1 2 8

Consider Placeholders As Dimensions…

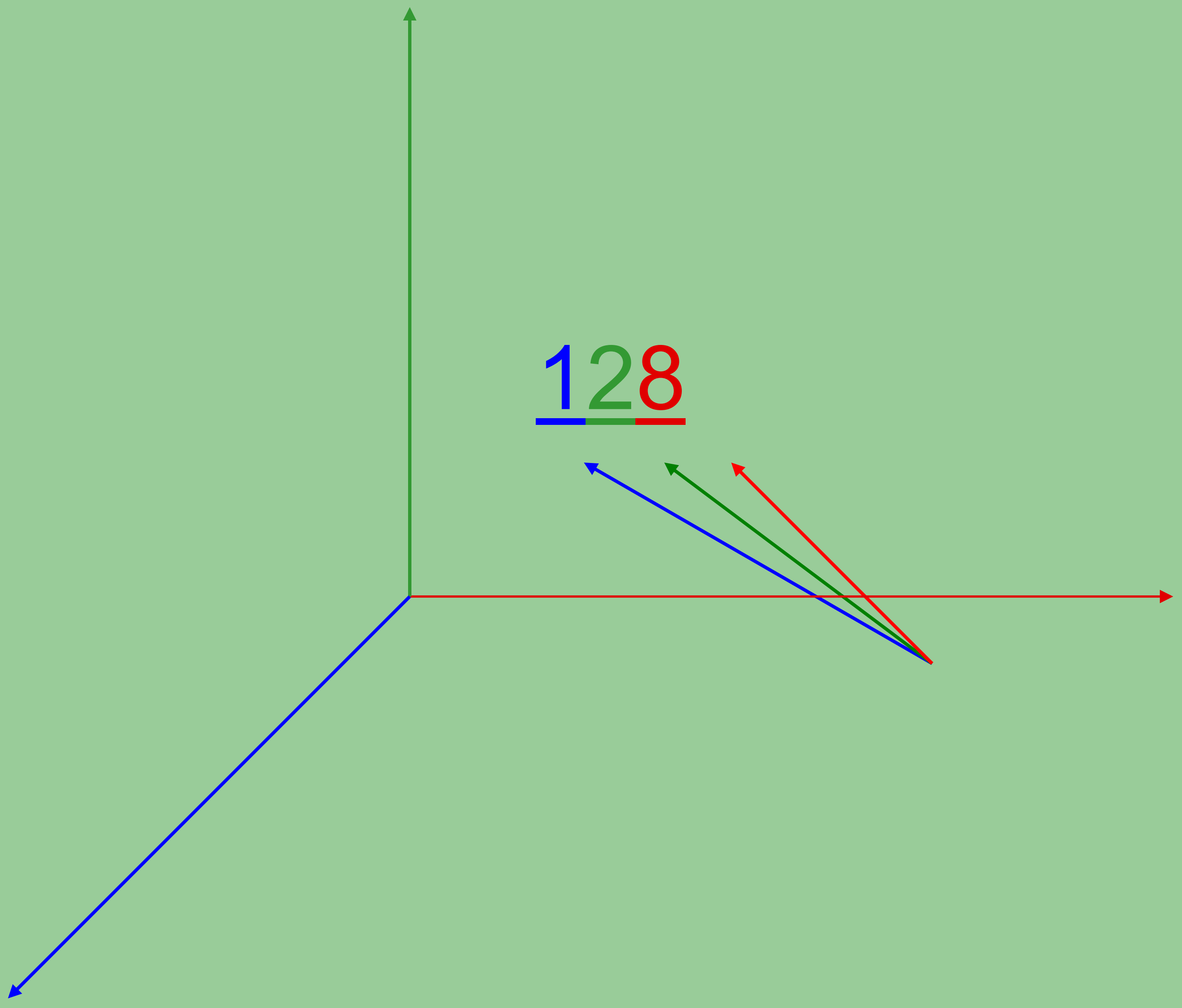
How many placeholders?

128

Three

128

Three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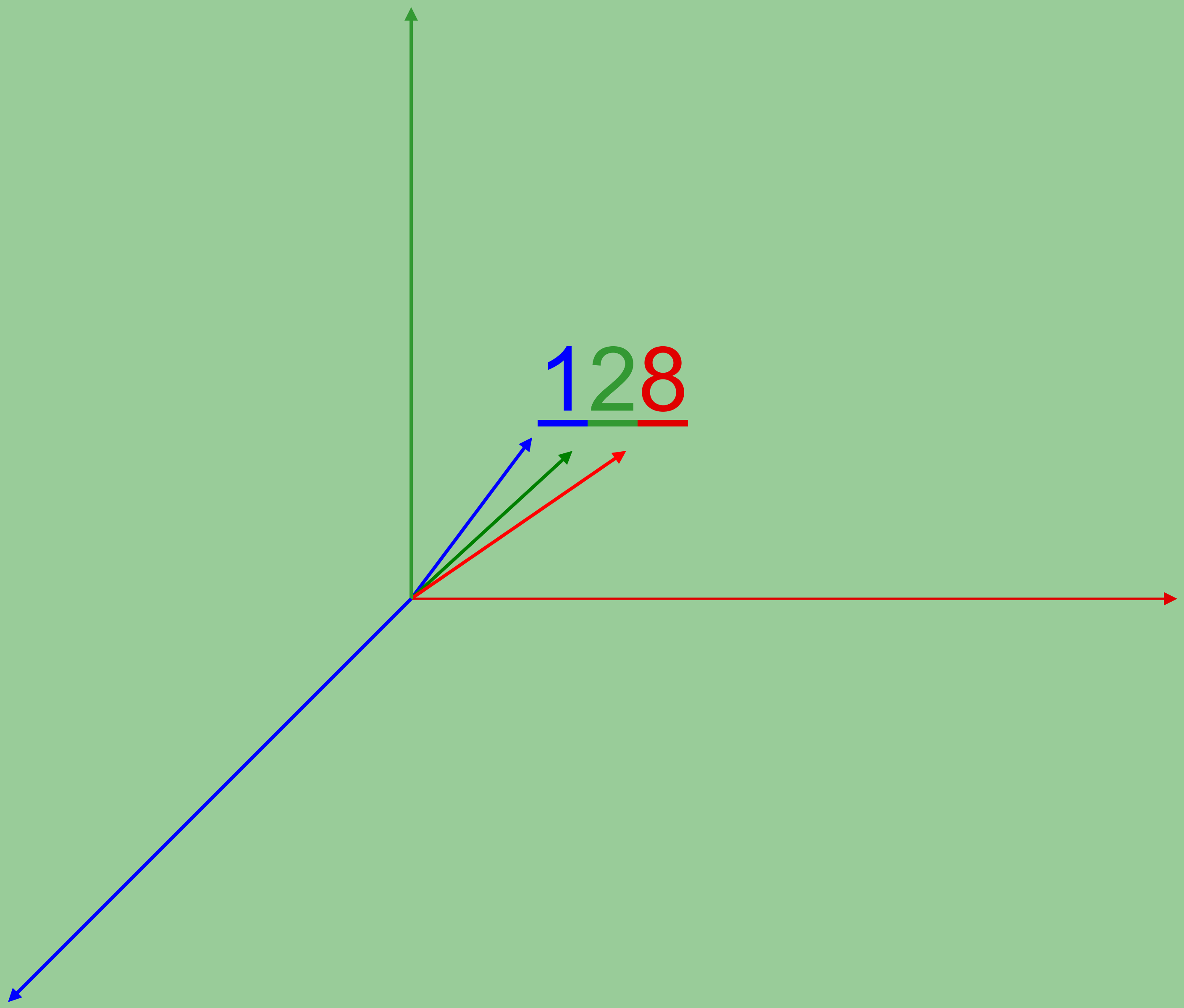
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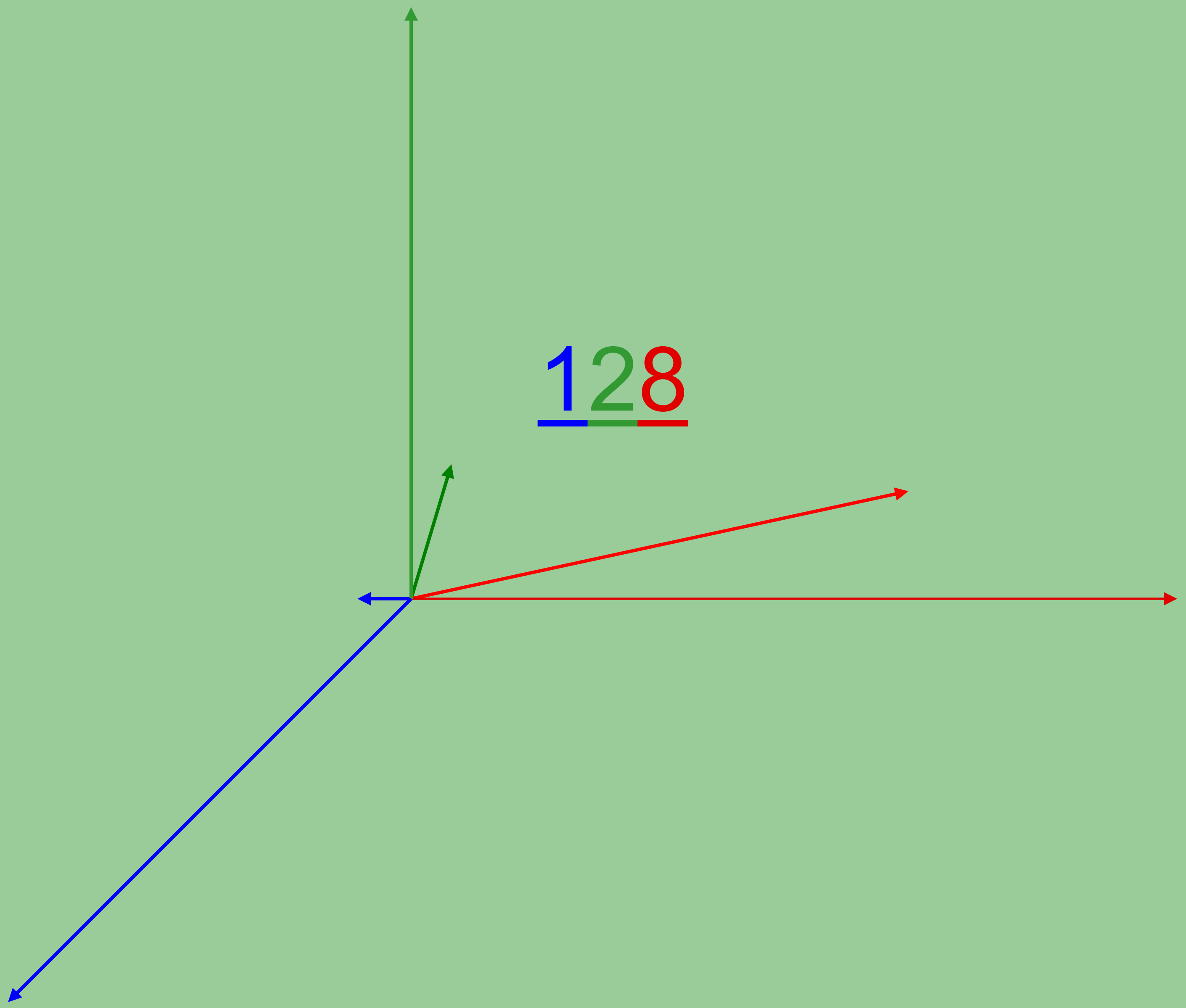
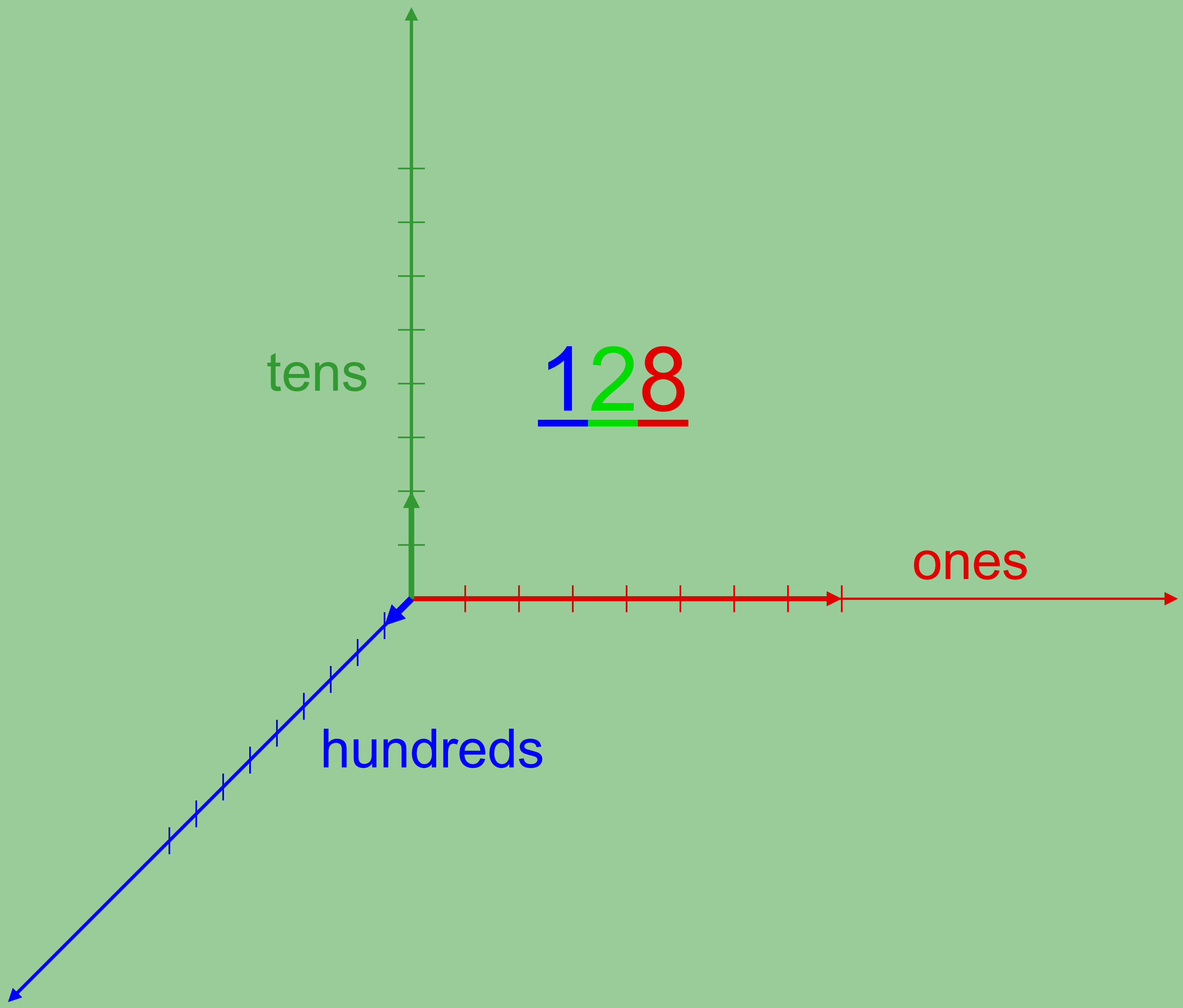
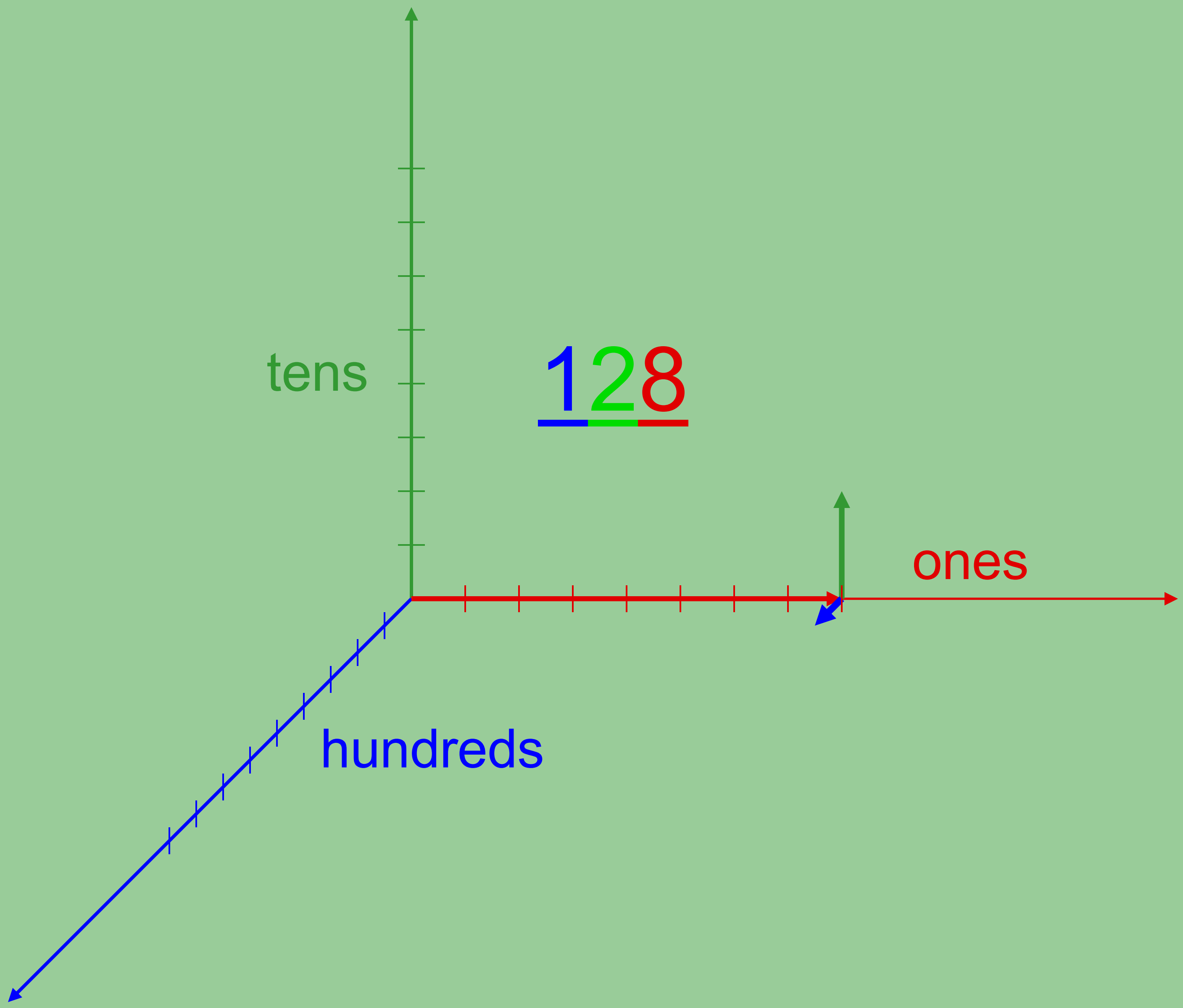
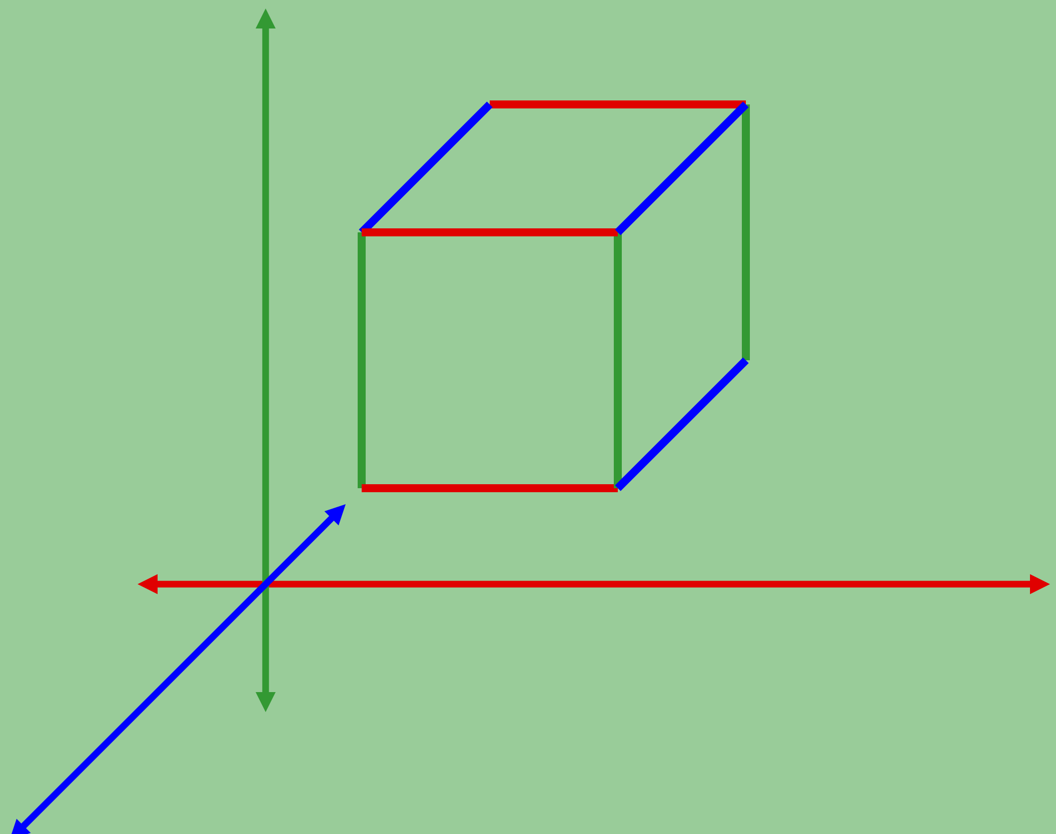

128


tens
128
ones
hundreds

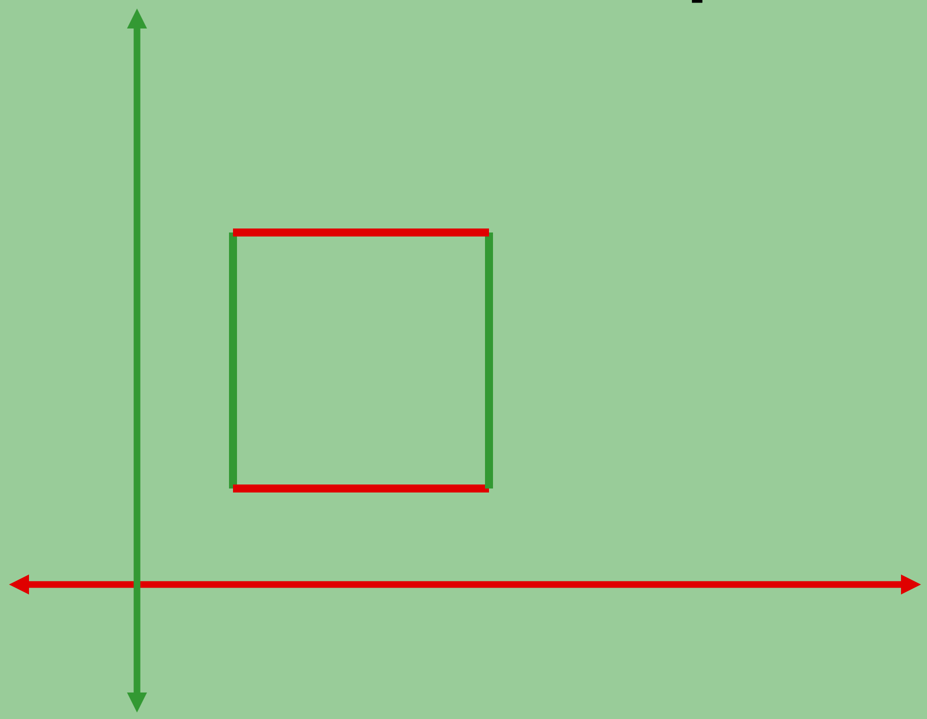


tens
128
ones
hundreds

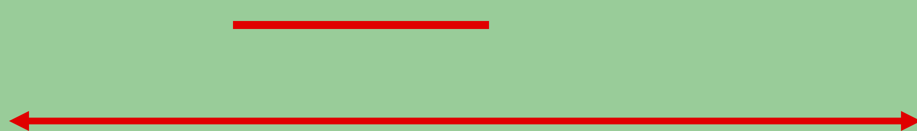
How many dimensions in a cube?



How many dimensions in a square?



How many dimensions in a line segment?



How many dimensions in a point?

How many dimensions in a point?

0

How many symbols in base one?

How many symbols in base one?

|

How do we count in base one?

Base 1 – Counting

|

||

||||

~~||||~~

How do we add in base one?

$$|| + ||| = ?$$

|| + ||| = ?

|| |||

|| + ||| = ?

|||||

|| + ||| =

|||||

$$|| + ||| = |||||$$

Addition

is equivalent to

Grouping

Grouping
is a powerful
idea.

Addition
is “closed”
in base one.

Meaning we can
represent any sum
in the same counting system.
(given sufficient *time*)

How do we subtract

In base one?

|| - ||| = ?

|| - ||| = ?

|| |||

|| - ||| = ?

|| |||

|| - ||| = ?

| ||

| |

|| - ||| = ?

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|| - ||| = ?

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|| - ||| = ?

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|| - ||| = ?

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||

|| - ||| = ?

|

| | - | | | = ? |

|| - ||| = |

Subtraction
is equivalent to
Separating

Subtraction
is not “closed”
in base one.

The *antistick*
requires another bit
that makes base two!

$|| - ||| = |$

The antistick is
a negative stick with
the annihilation property:

| + | =

How do we multiply
In base one?

$|||| \ x \ || \ =$

The second number
tells us the
number of copies
of the first.

|||| x || = ||||
||||

And visa versa

|| x |||| = || || || ||

This demonstrates that multiplication in base one is commutative.

Multiplication is repeated addition.

Multiplication
in base one
is “closed”.

Base One Squares

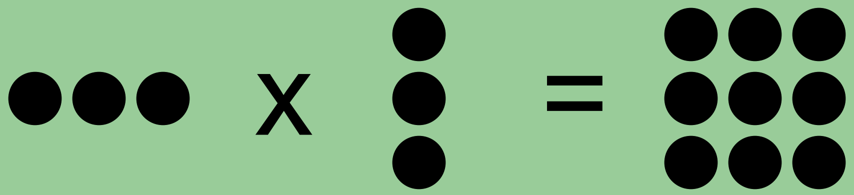
Squaring is
a special case
of multiplication.

|||

||| x ||| = |||

|||

Choosing a different symbol reminds us of the meaning of “squaring”.


x
=

Or:

$$\begin{array}{ccc}\square & \square & \square\end{array} \times \begin{array}{c}\square \\ \square \\ \square\end{array} = \begin{array}{ccc}\square & \square & \square \\ \square & \square & \square \\ \square & \square & \square\end{array}$$

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□□□ □□□ □□□
□□□ □□□ □□□

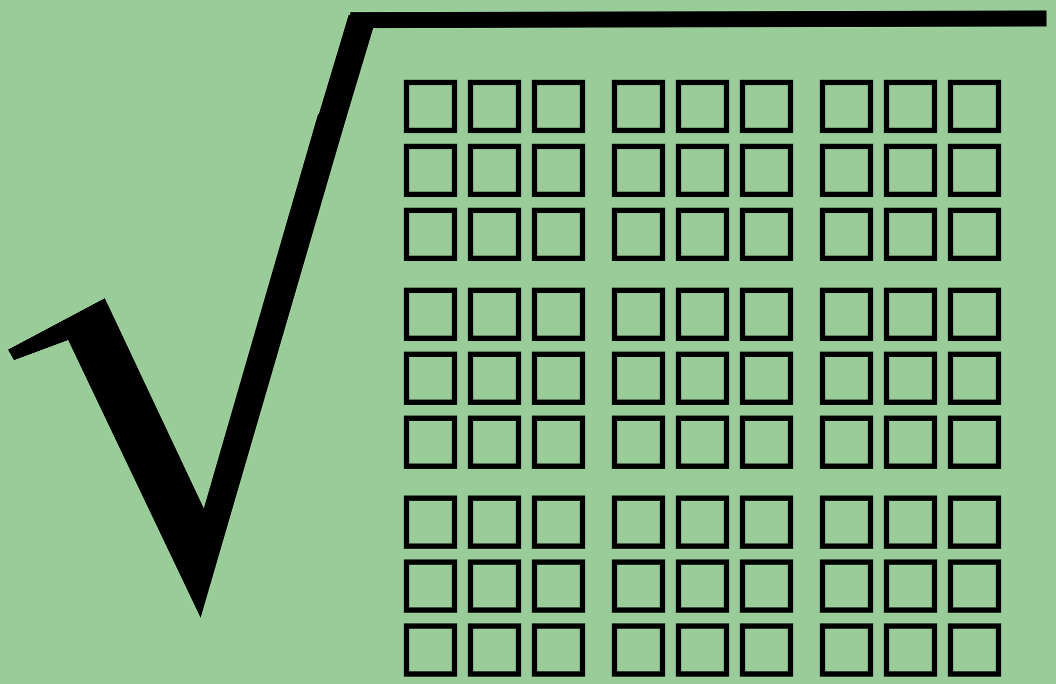
□□□   □□□   □□□ □□□ □□□
□□□ x □□□ = □□□ □□□ □□□
□□□   □□□   □□□ □□□ □□□

□□□ □□□ □□□
□□□ □□□ □□□
□□□ □□□ □□□

How do we find the square root in base one?

“Find the number that when multiplied by itself gives the original number.”

If the number is
a perfect square,
the square root is
the length of one side.



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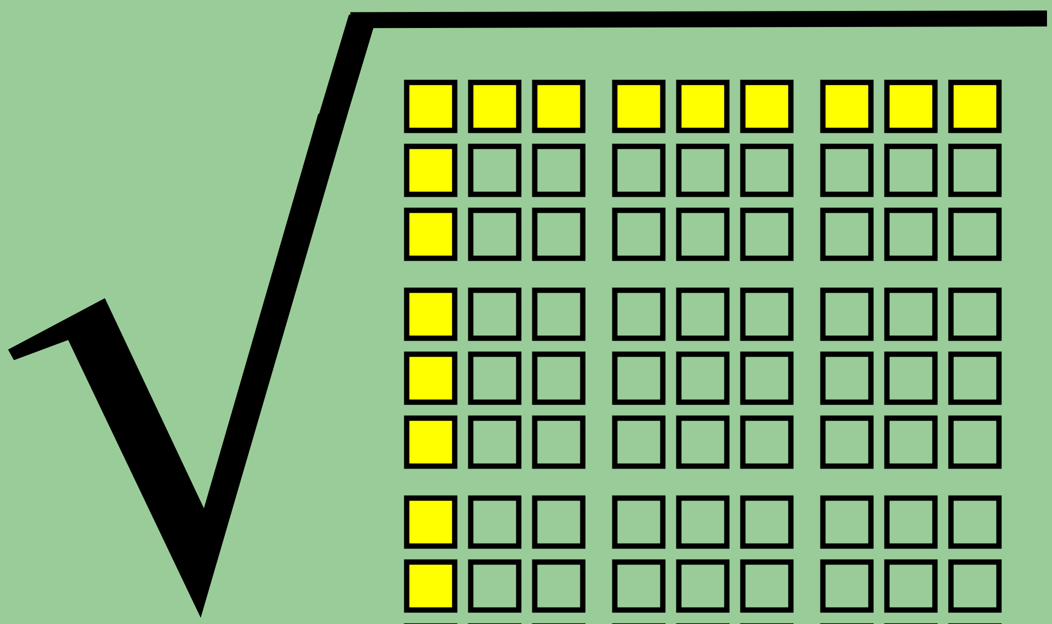

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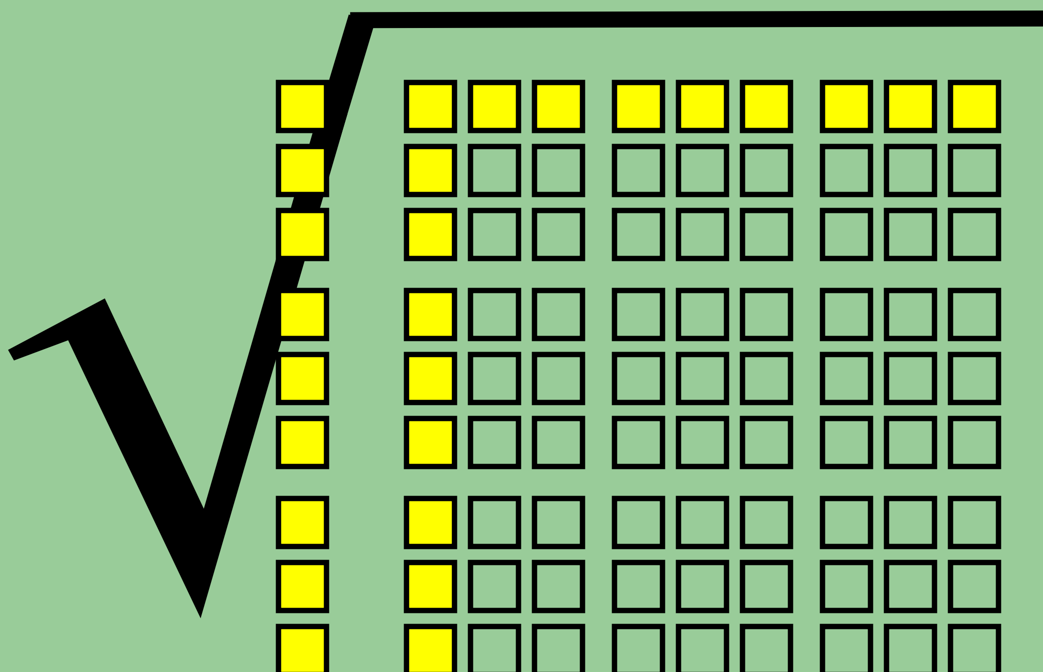

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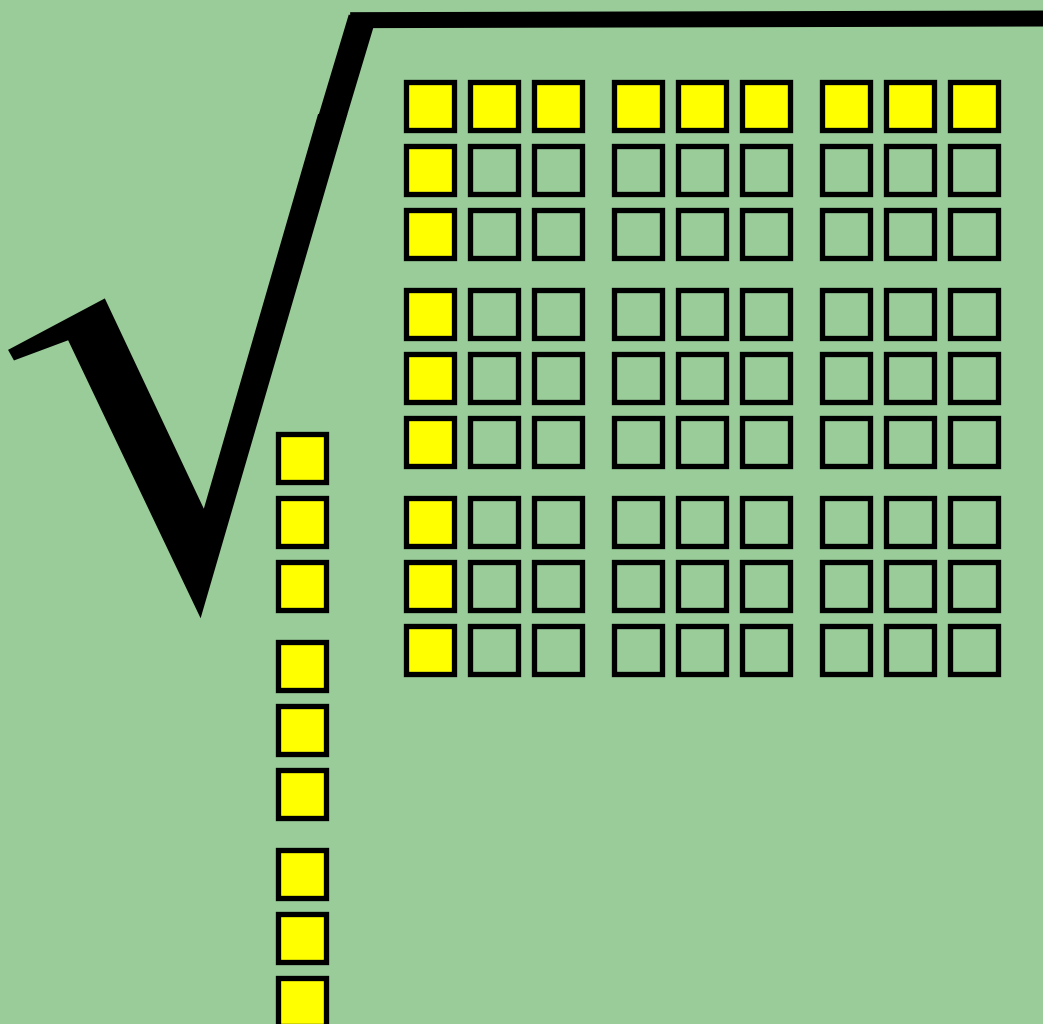
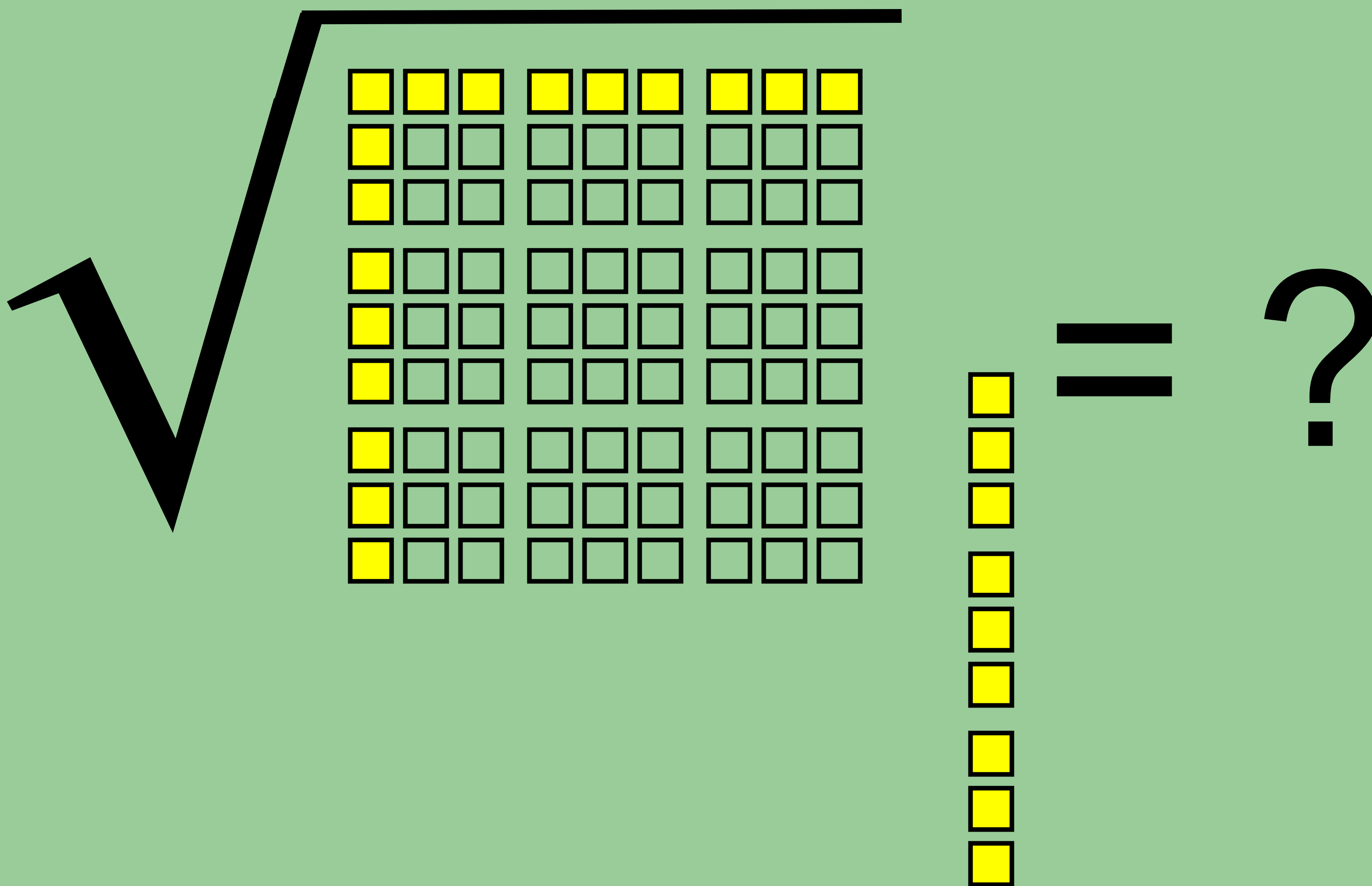
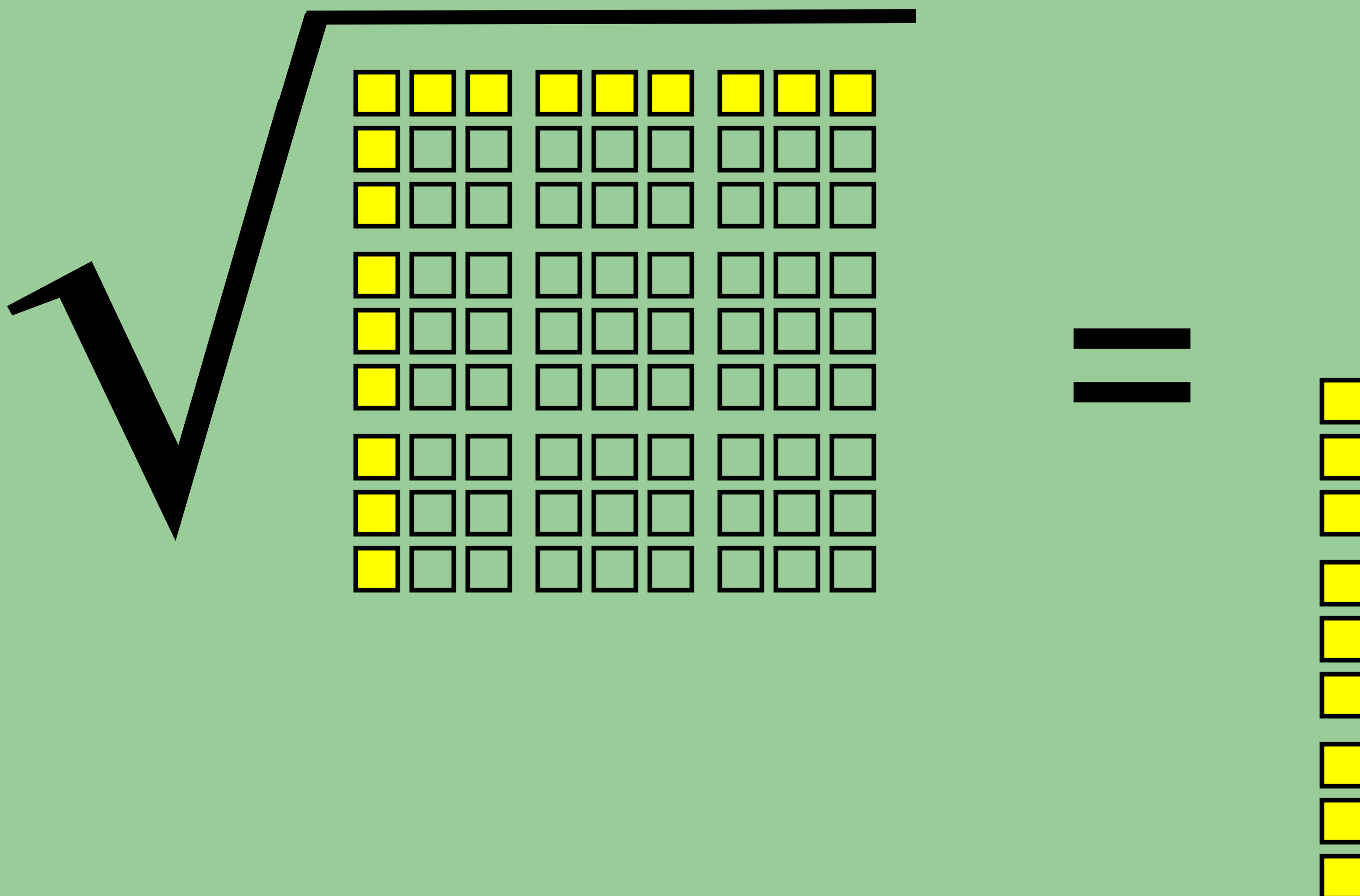
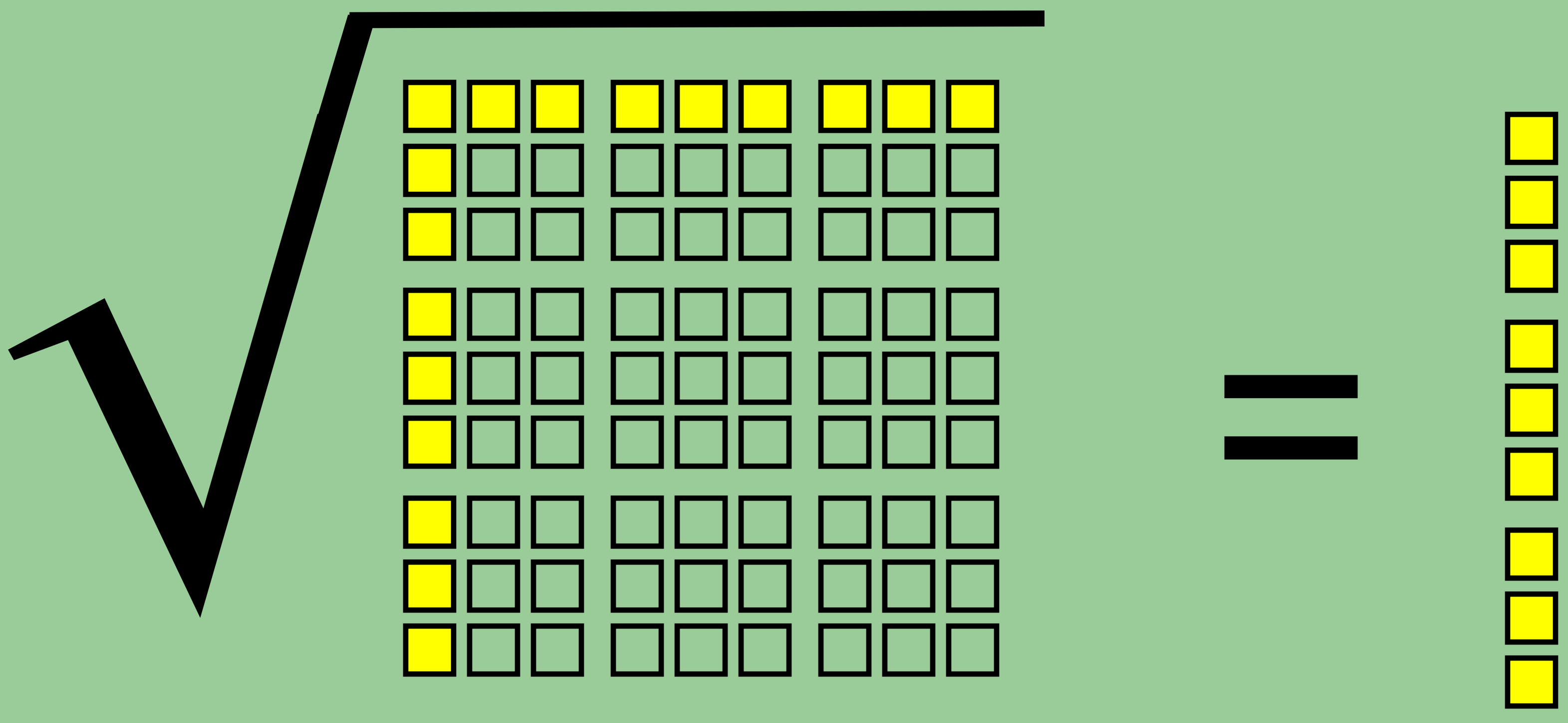


= ?







How do we divide
In base one?

Division

is repeated

subtraction.

$$\frac{\begin{matrix}\square\square\square\\ \square\square\square\\ \square\square\square\end{matrix}}{\square\square\square} = ?$$

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= ?

$$\frac{\text{□□□ □□□ □□□}}{\text{□□□}} = ?$$

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□ □

= ?

$$\frac{□□□\ □□□\ □□□}{□□□} = ?$$

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□□□

= ?

□□□

Division, like subtraction
is not closed
in base one.

$$\frac{\square\square\square}{\begin{matrix}\square\square\square\\ \square\square\square\\ \square\square\square\end{matrix}} = ?$$

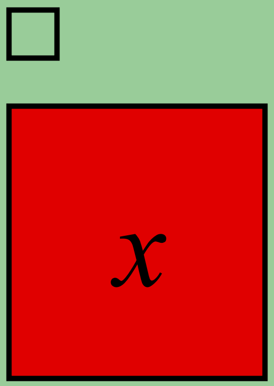
$$\frac{\square\square\square}{\begin{matrix}\square\square\square\\ \square\square\square\\ \square\square\square\end{matrix}} = \frac{\square}{\square\square\square}$$

Base One Trigonometry

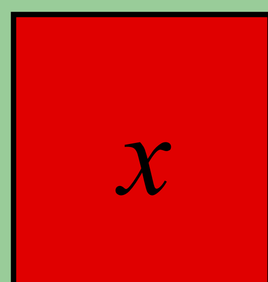
We must introduce some new symbols.

These symbols
are not used for counting.

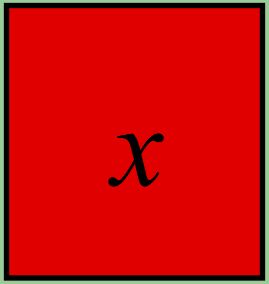
These symbols are containers for copies of our counting symbol.


x

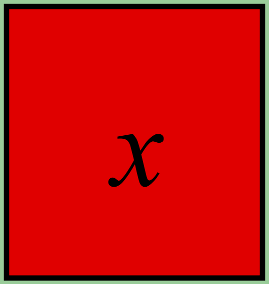
□□

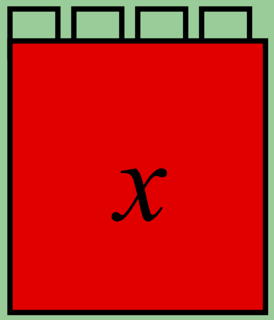

x

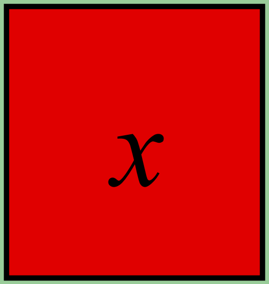
□□□


x

□□□□


x


x

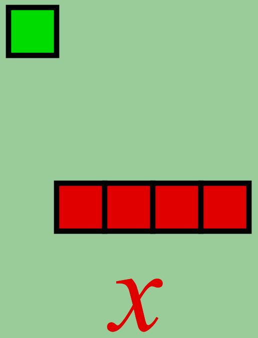

x

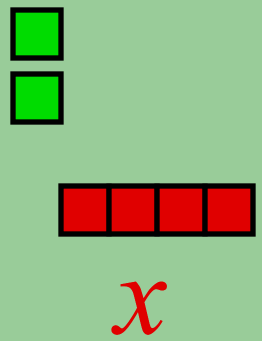
x

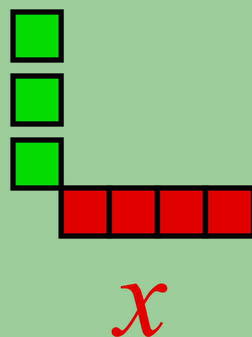
x

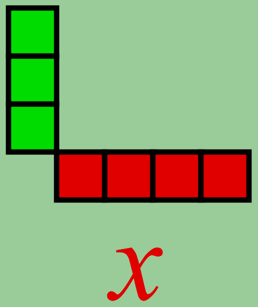
x

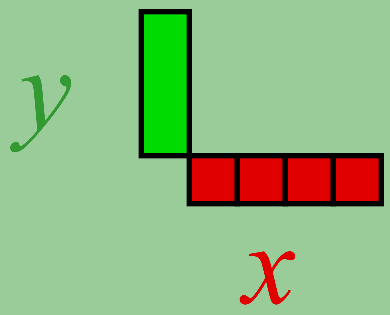
x

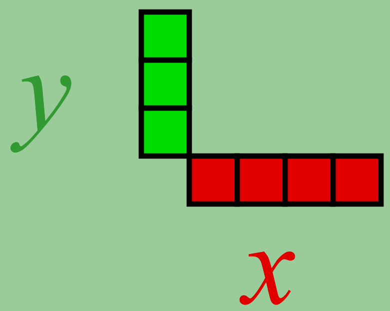

x


x


x


x


y
x


y
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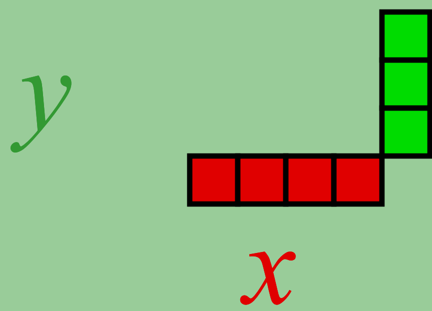
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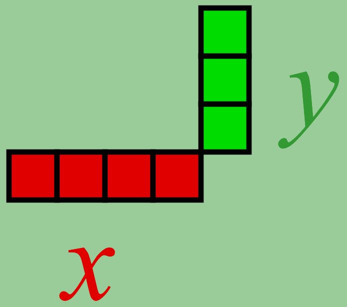
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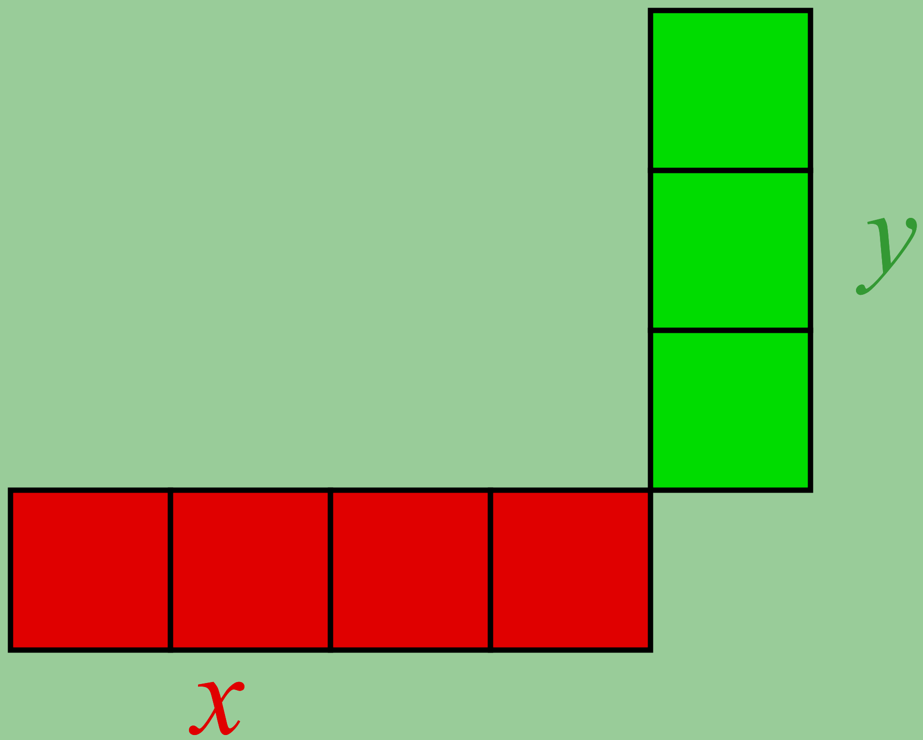

y
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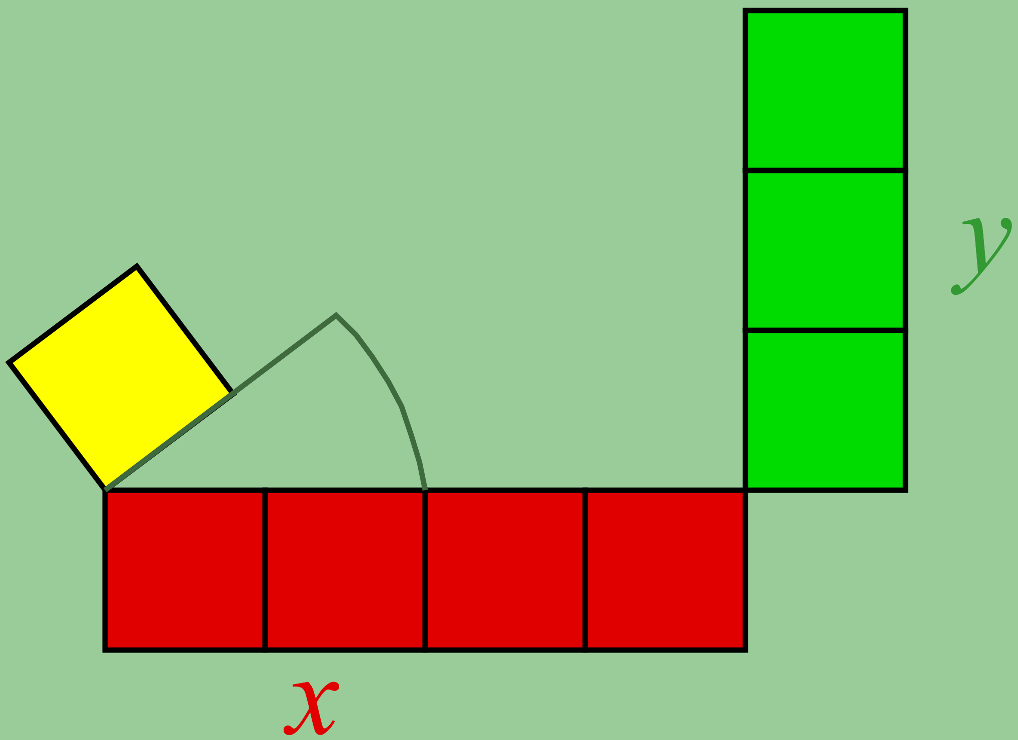
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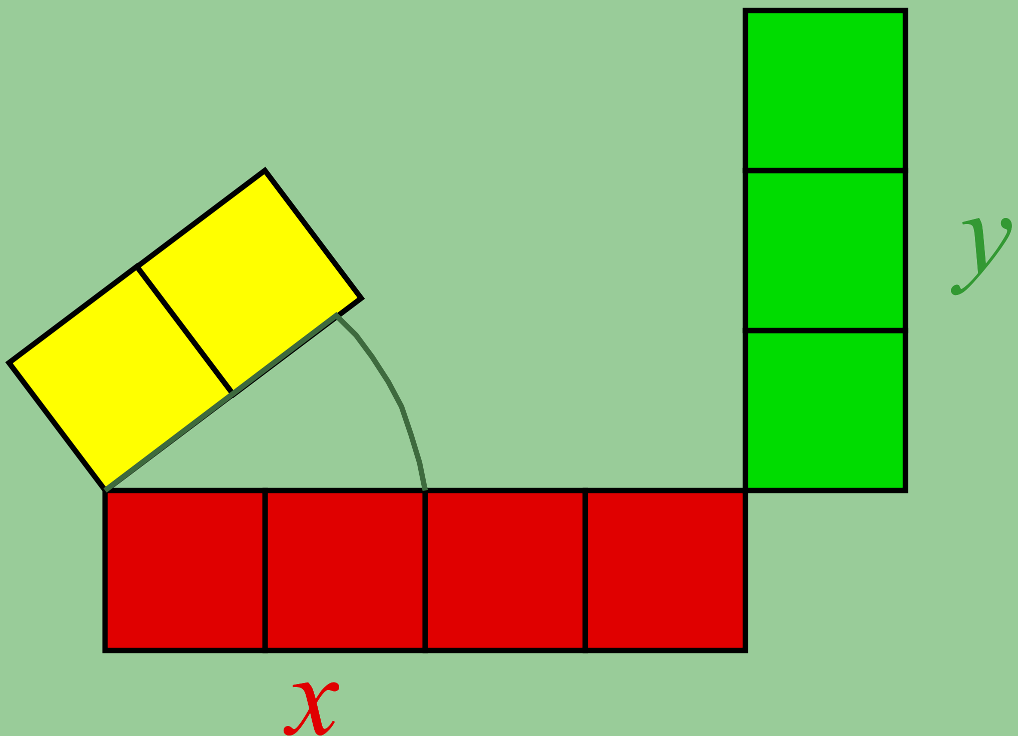
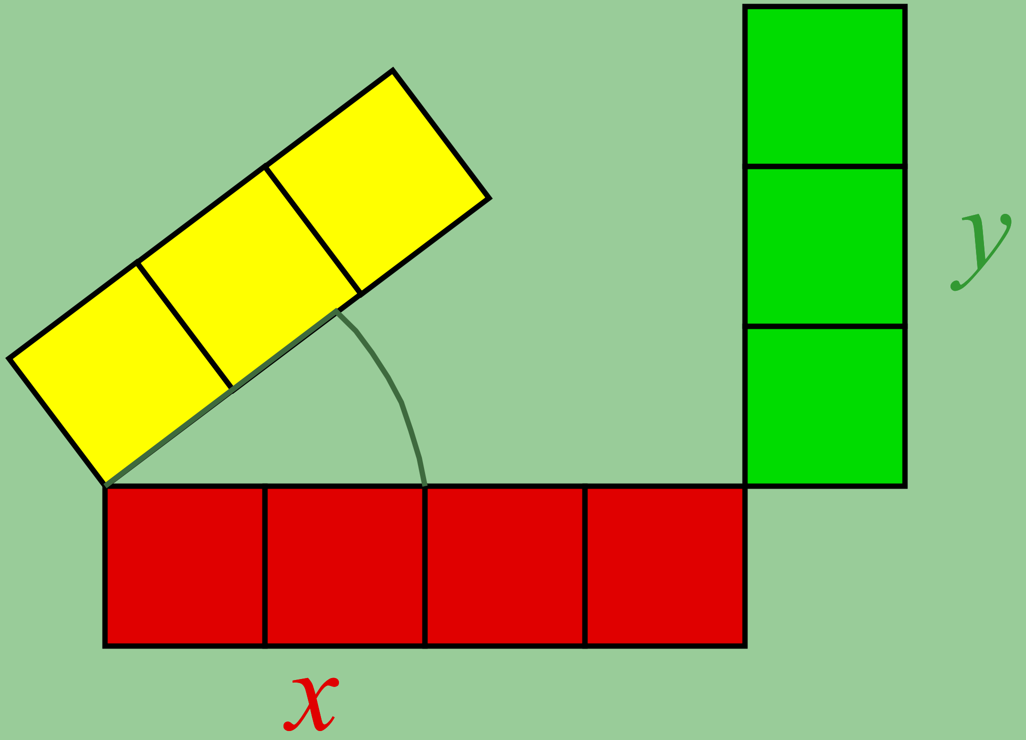
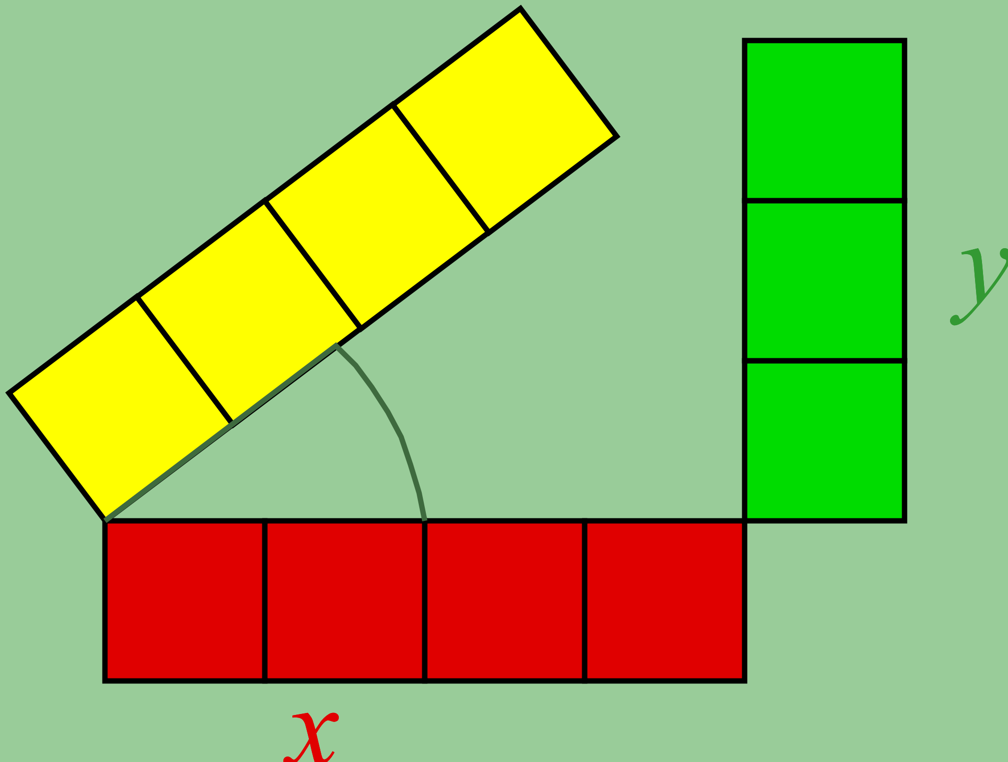
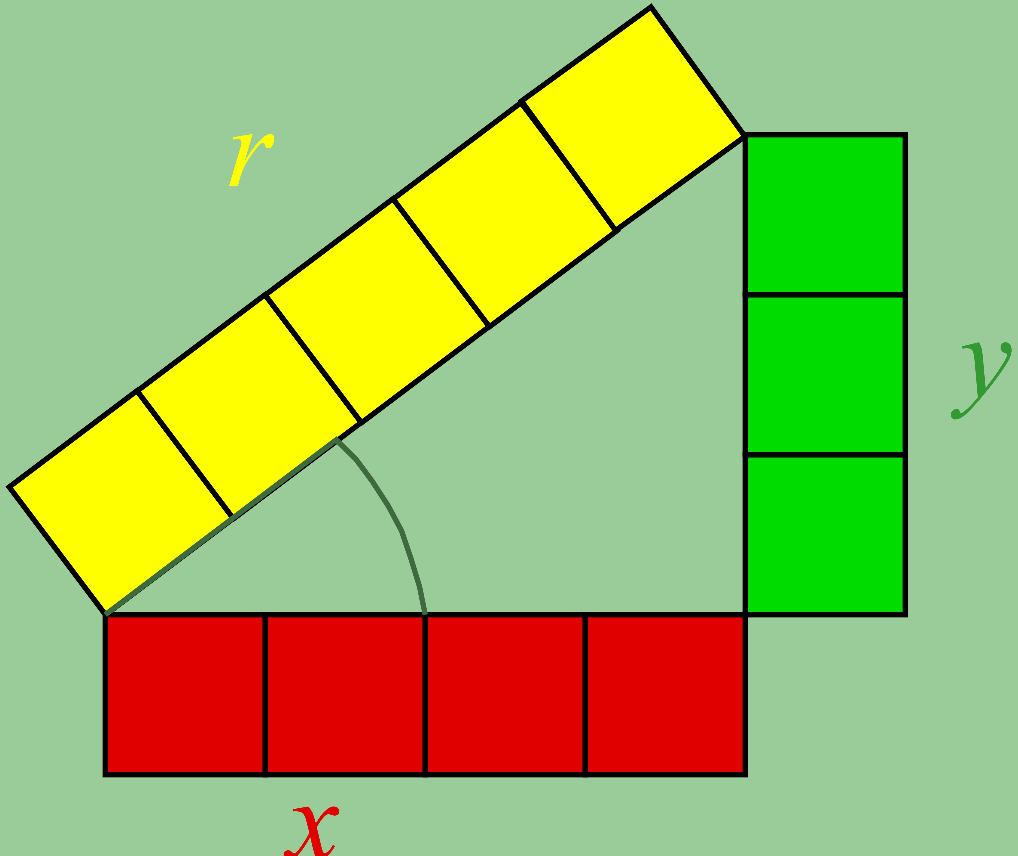
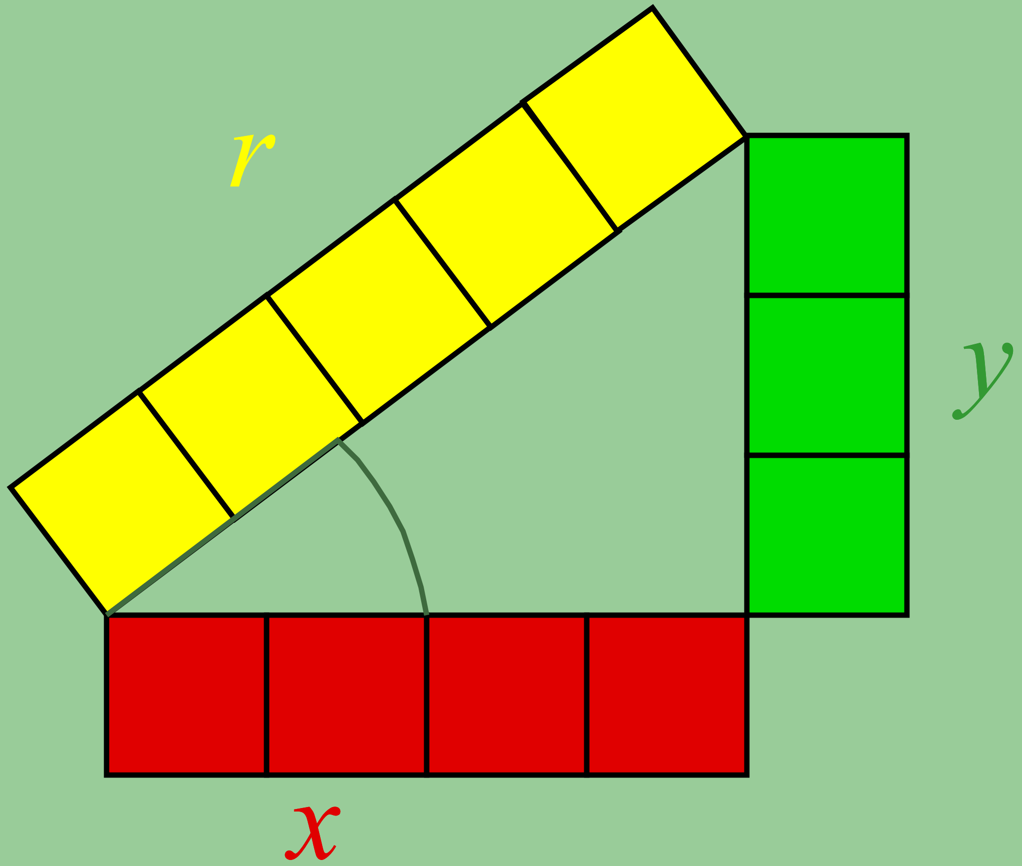

y
x



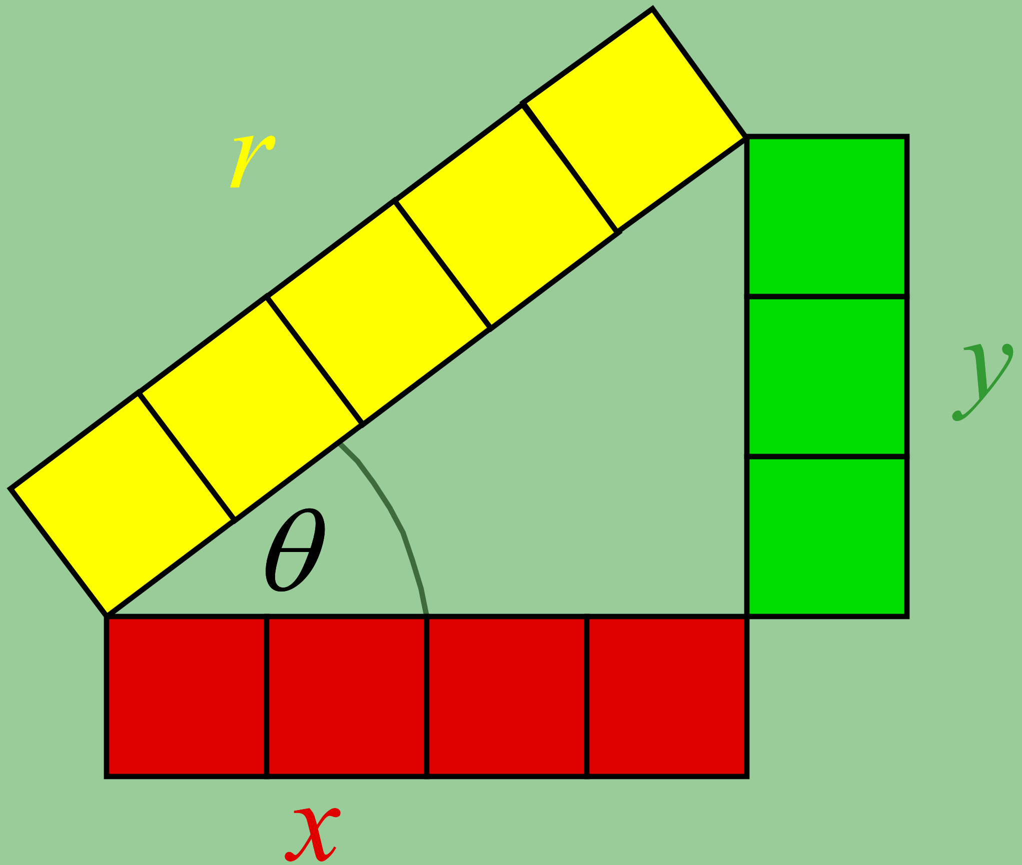
y
x


y
x

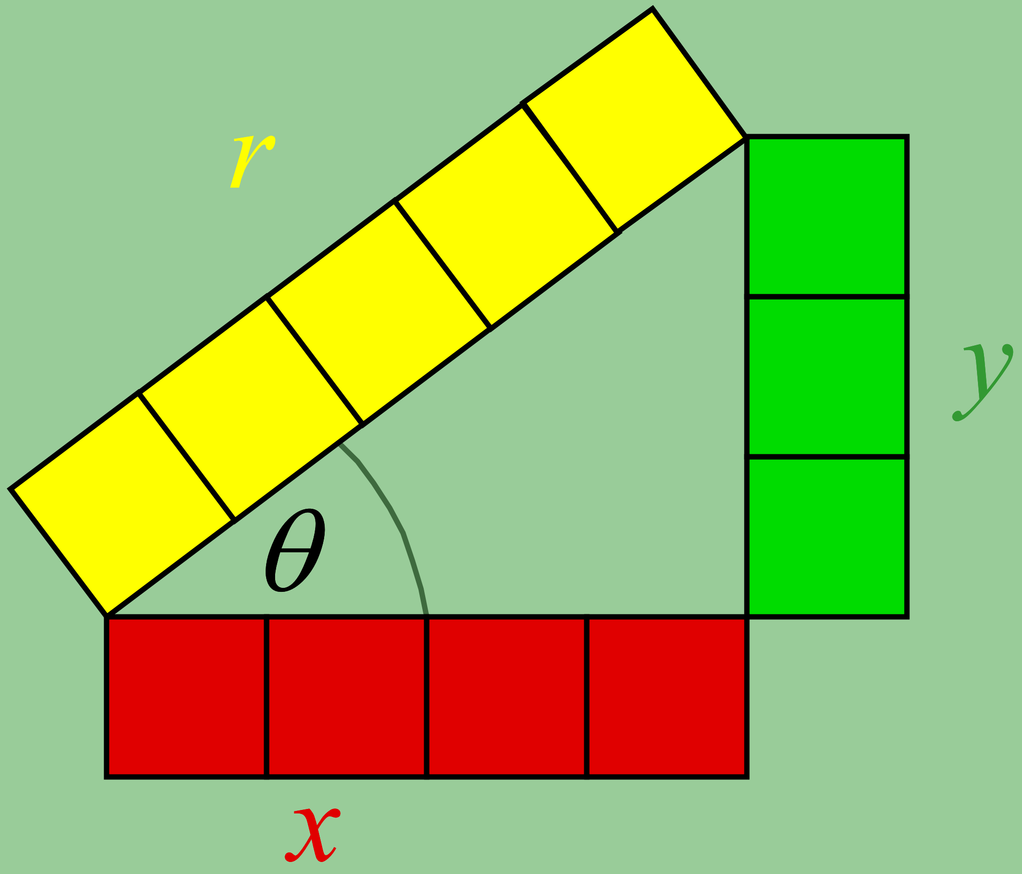

r
y
x


r
y
x

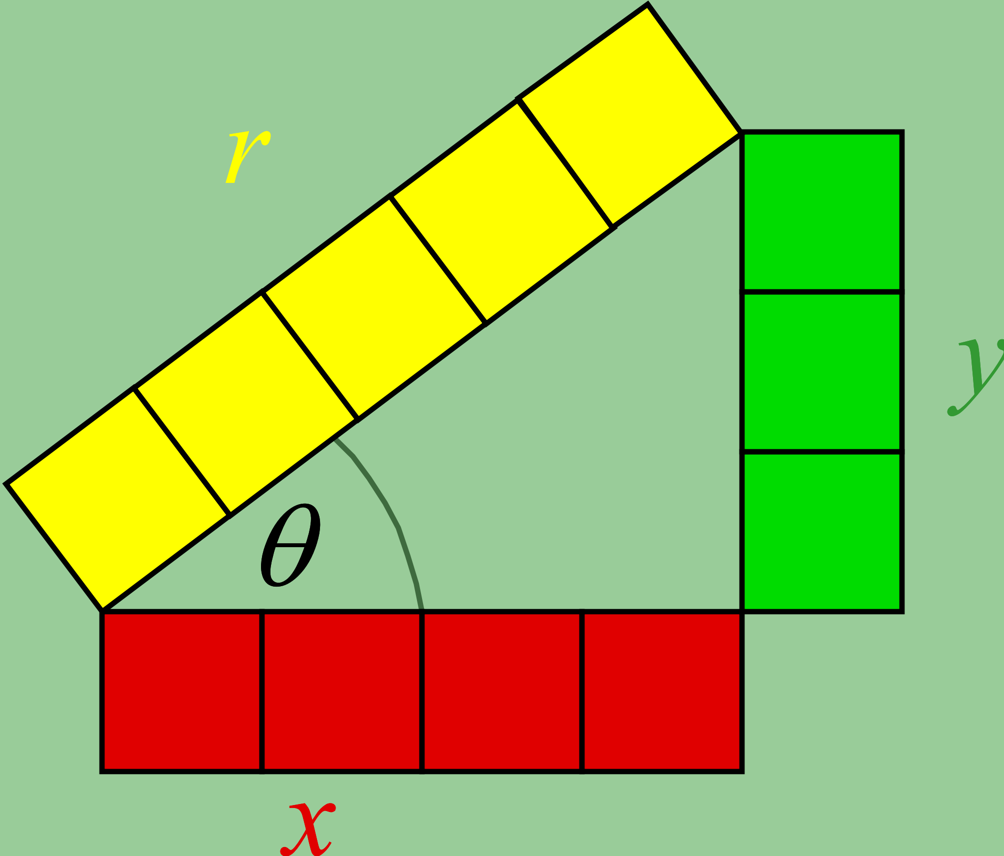
Name relationships.



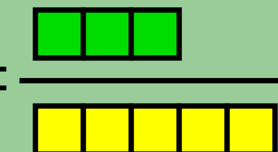
r
y
θ
x

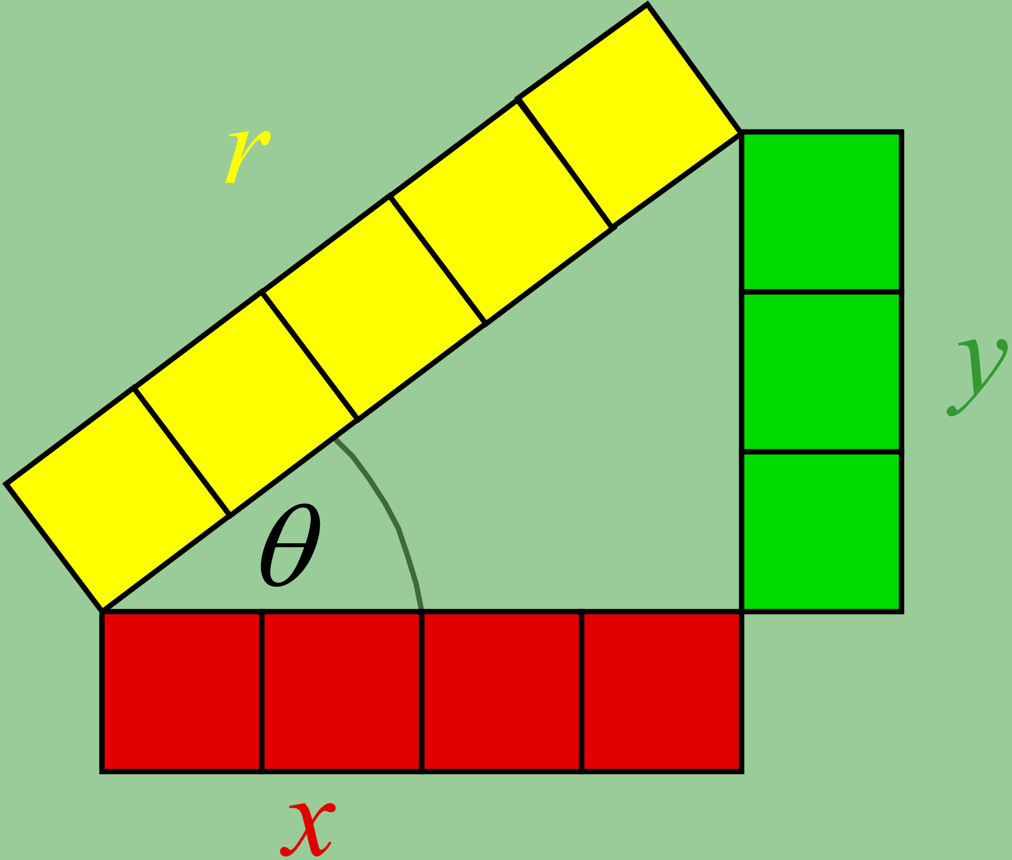


$$\sin(\theta) = \frac{y}{r}$$

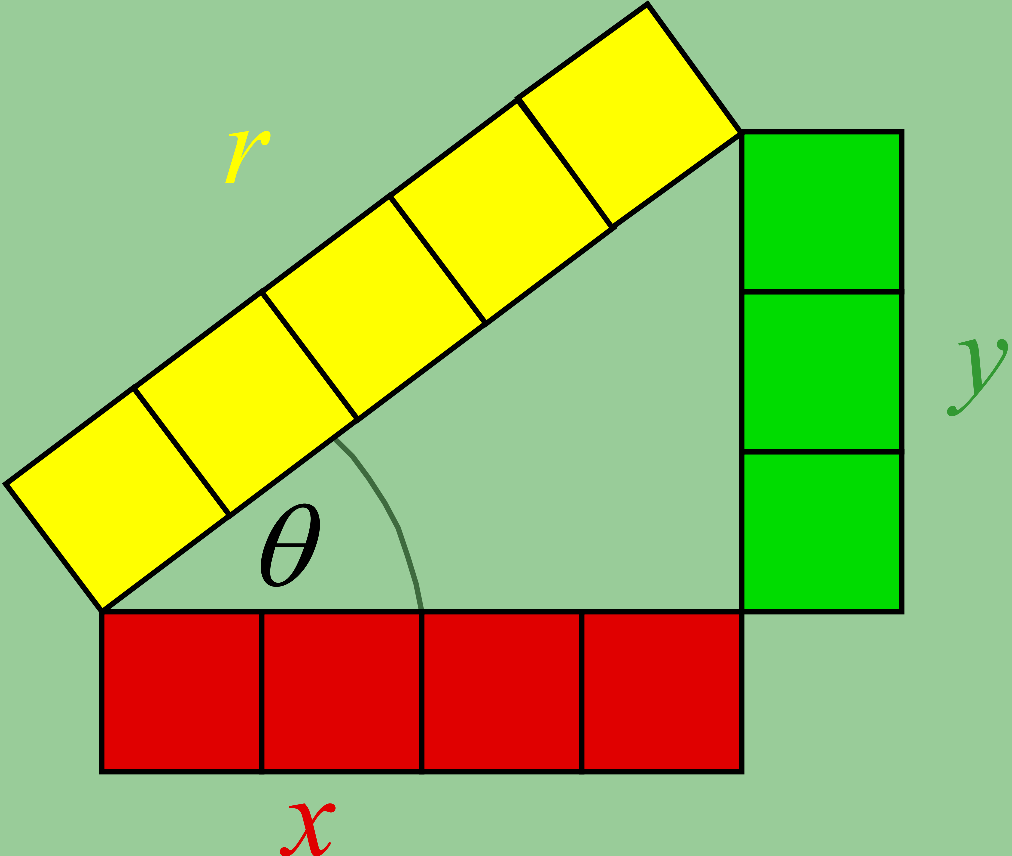


$$\sin(\theta) = \frac{y}{r}$$

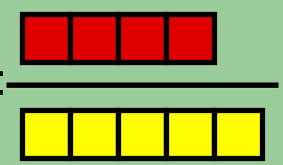
$\sin(\theta) =$ 

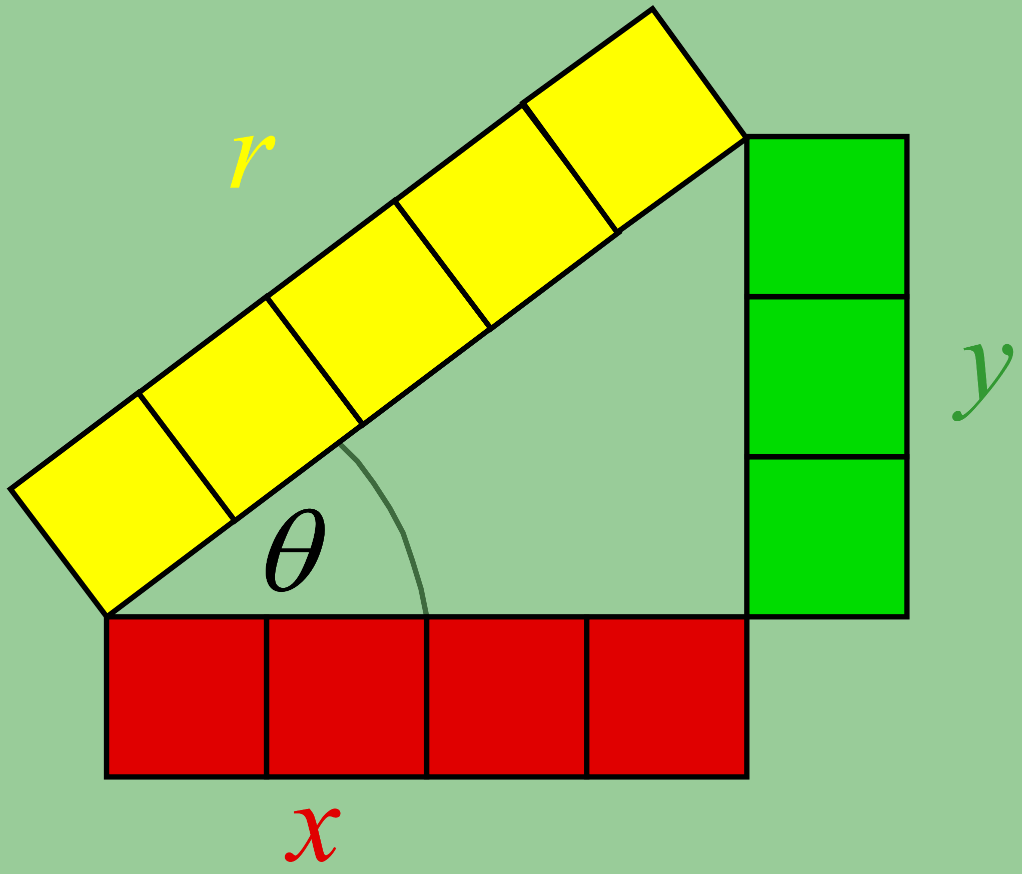


$$\cos(\theta) = \frac{x}{r}$$

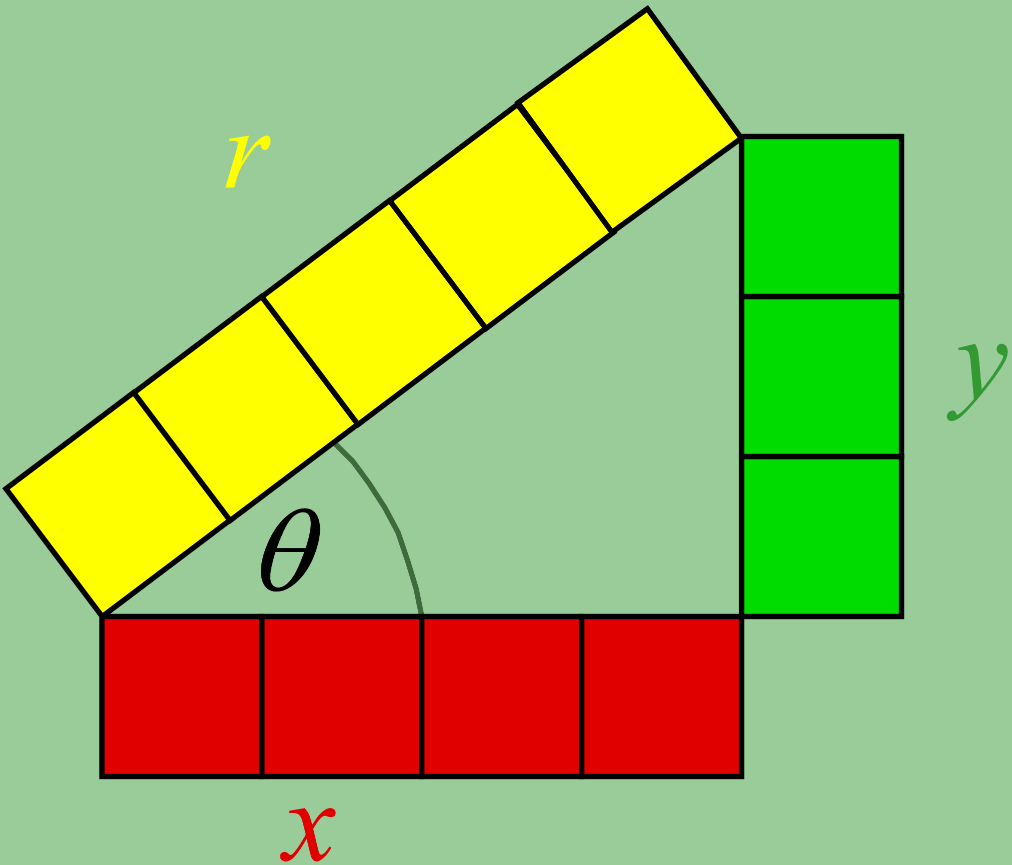


$$\cos(\theta) = \frac{x}{r}$$

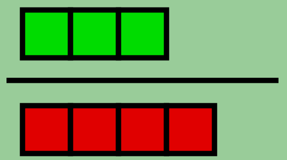
$\cos(\theta) =$ 



$$\tan(\theta) = \frac{y}{x}$$



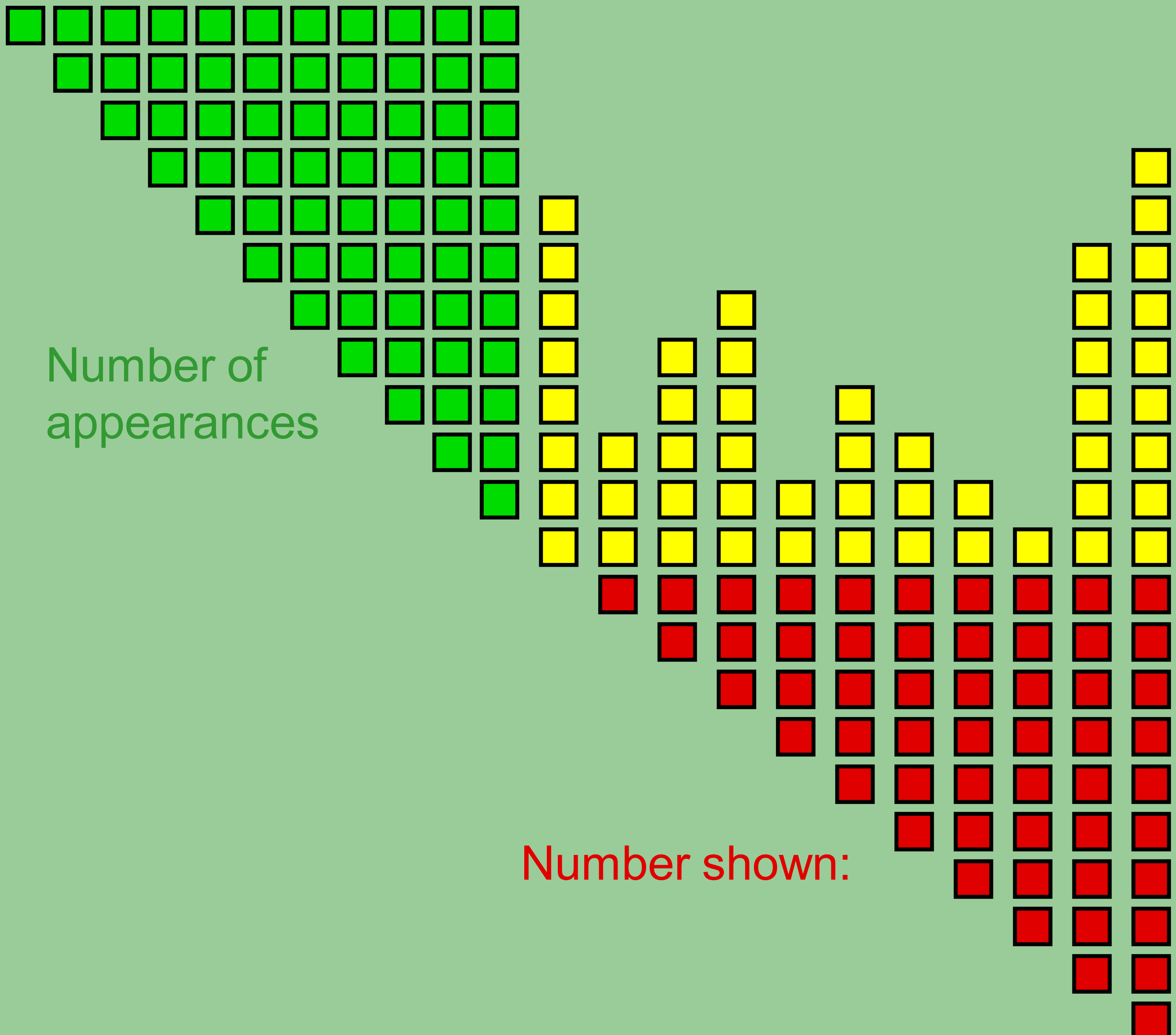
$$\tan(\theta) = \frac{y}{x}$$

$\tan(\theta) =$ 

Base One Probability

Ask some friends
to show you a number
by holding up their hands.

Then plot the result in base one.


Number of appearances
Number shown:

Questions?