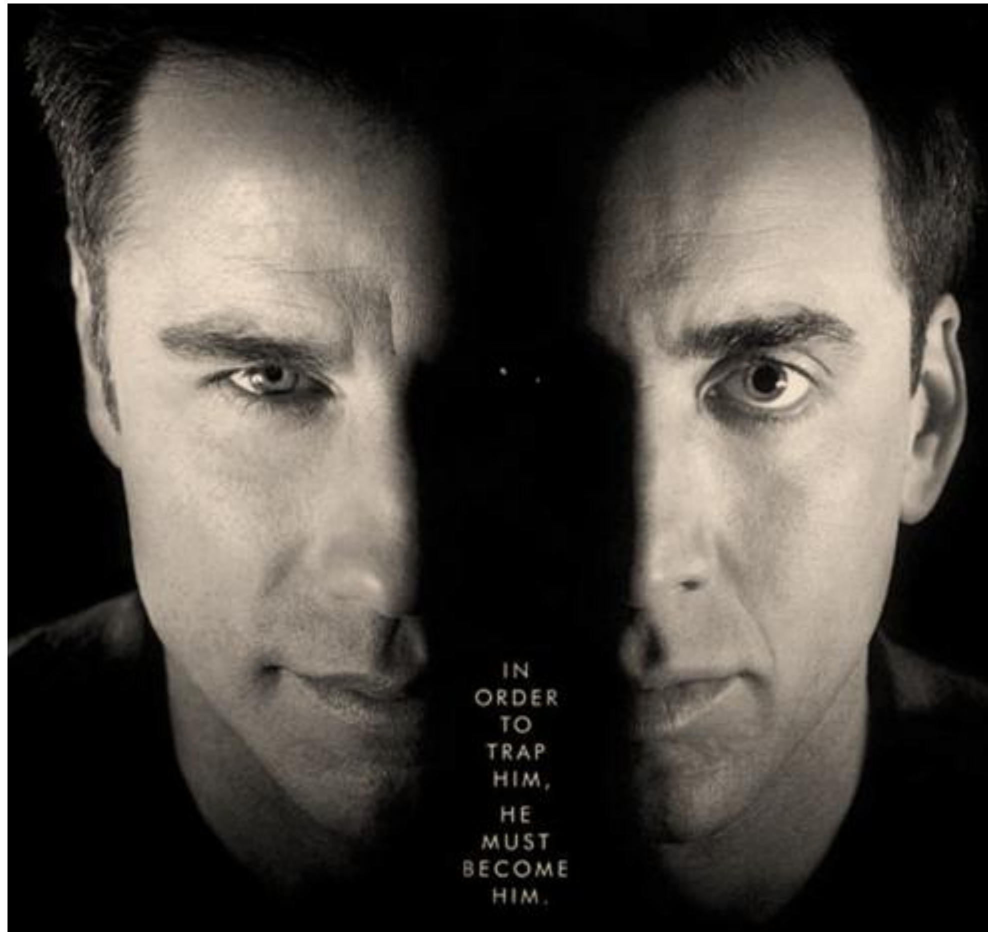
Learning Calculus With Geometry Expressions™

by L. Van Warren



- Touchstone Pictures

Lecture 24:
Integration By Substitution

Chapter 6: Integration Techniques

Lecture	Topic
23	Integration: Cartesian and Polar
24	**Integration by Substitution**
25	Integration by Parts
26	Area Between Curves

Calculus Inspiration

Hermann Minkowsi

PhD. Königsberg 1885
Göttingen University

- French Prize 1883 while student
- Minkowski Space
- Unified Space and Time
- Taught Einstein
- Friend of David Hilbert
- Died at 44 of Appendicitis

C. F. L. Lindemann (47)

C. Felix Klein (57)

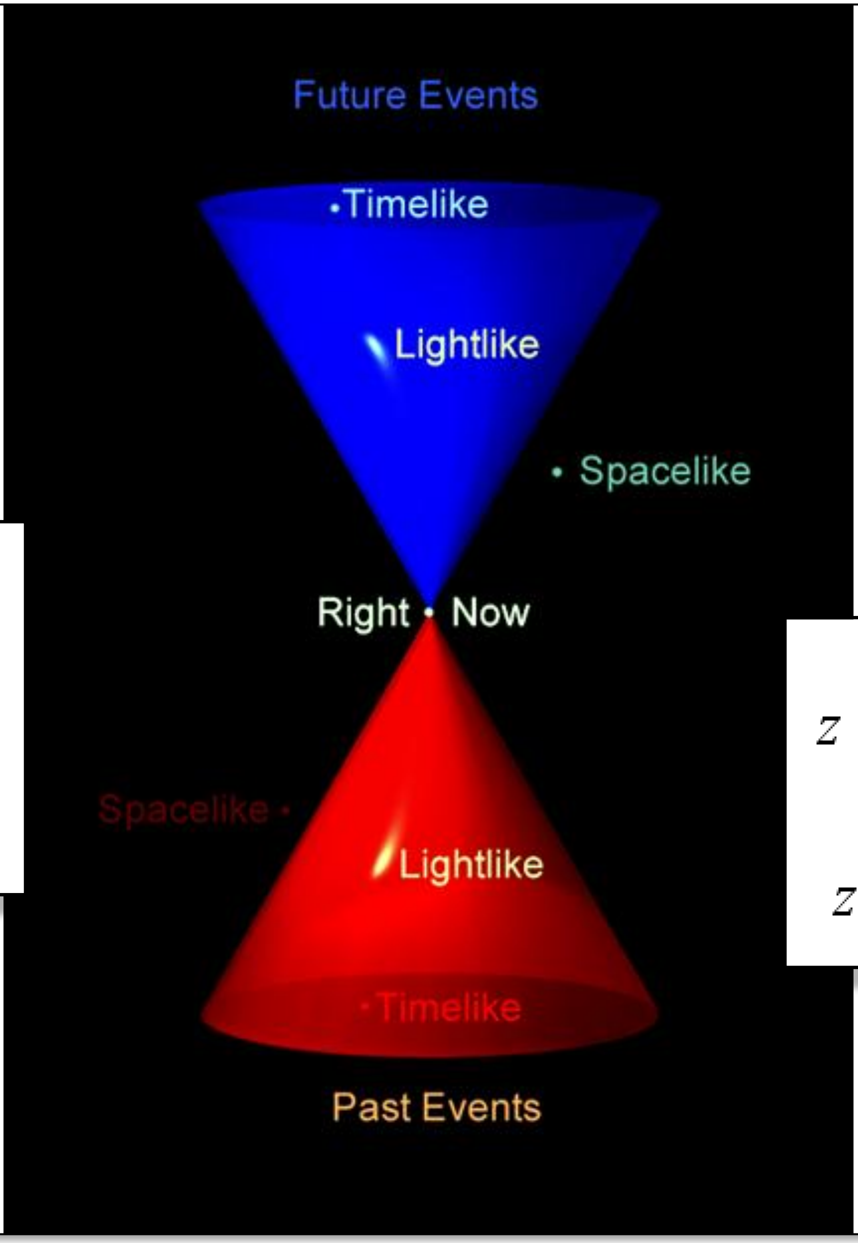
Julius Plücker

Christian Gerling

Carl F. Gauß (8)

Spacelike Timelike and Lightlike:

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$z = \sqrt{x^2 + y^2}$$

$$z^2 = x^2 + y^2$$

Substitution 101:

A powerful symbolic trick:

$$\int f(u)du = \int f(g(x))\,g'(x)\,dx$$

Lecture24-Substitution101.gx

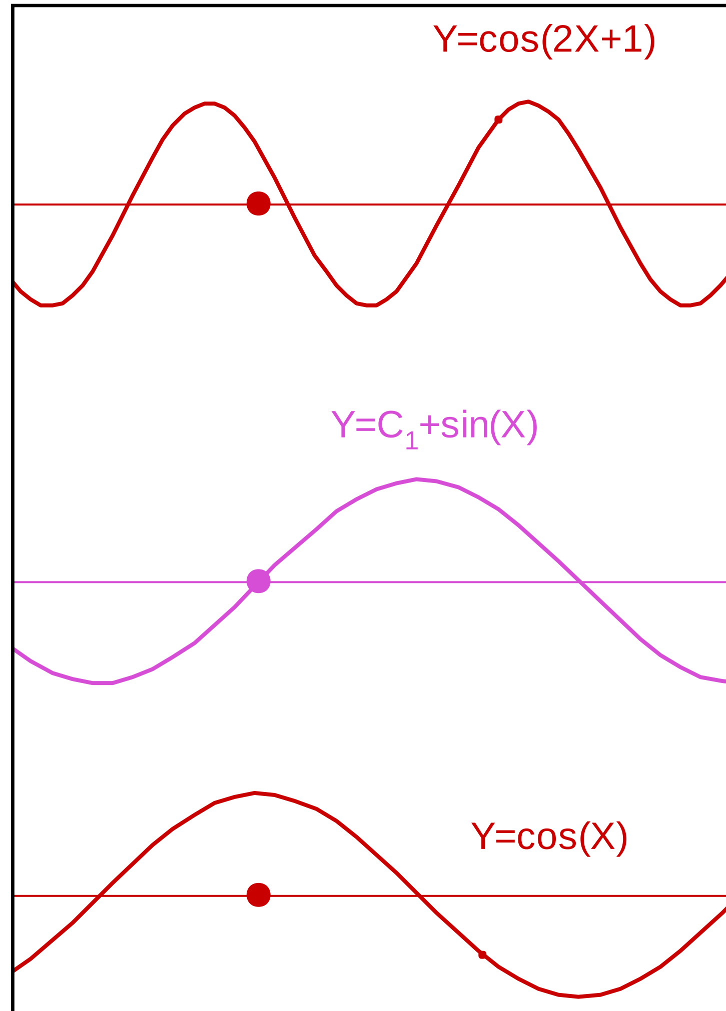
Substitution 101:

For example, we know that:

$$\int cos(x)\,dx = sin(x) + C_1\ ;$$

but what is:

$$\int cos(2x+1)\,dx = \ ?\ ;$$



Substitution 101:

For example, we know that:

$$\int \cos(x)\,dx = \sin(x) + C_1 \;;$$

but what is:

$$\int \cos(2x+1)\,dx = \;?\;;$$

Use substitution:

$$\textit{Let} \quad u = 2x+1$$

$$\textit{Then} \quad du = 2\,dx$$

Because

$$\frac{d}{dx}u = \frac{d}{dx}(2x+1) = 2$$

$$\textit{AND IF} \quad \frac{du}{dx} = 2$$

THEN

$$dx = \frac{du}{2}$$

Substituting we have:

$$\int \cos(u)\frac{du}{2} = \frac{1}{2}\int \cos(u)\,du$$

$$= \frac{1}{2}\sin(2x+1) + C_2$$

Substitution 101:

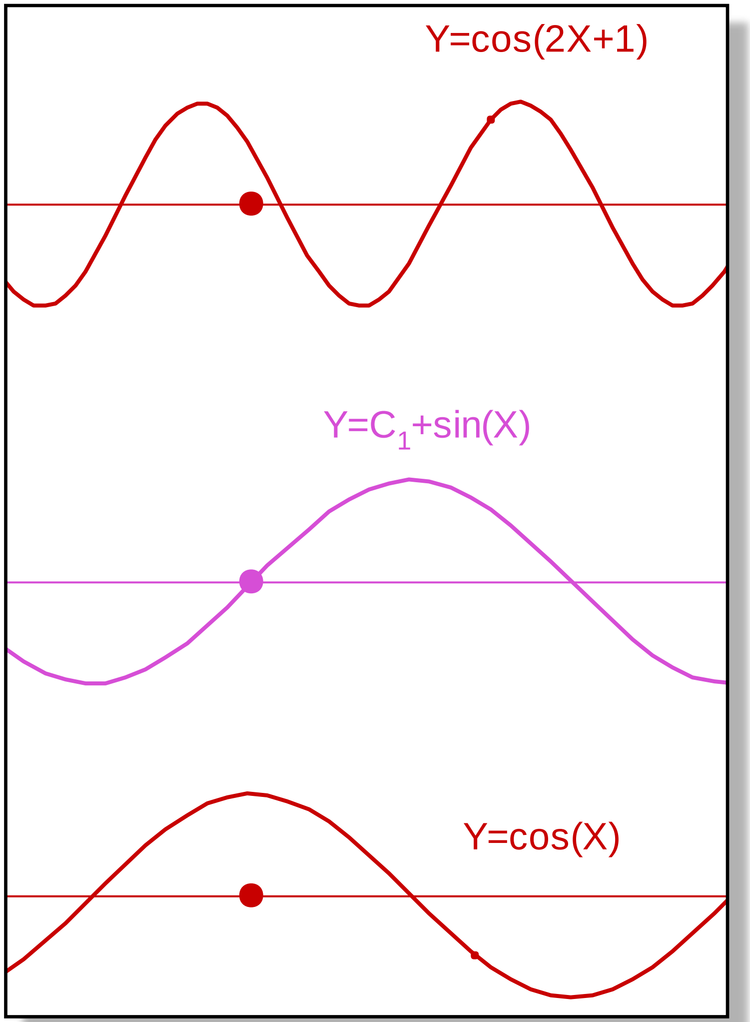
Lecture24-Substitution101.gx

Exercise:

Fill in the missing fourth curve which solves:

$$\int cos(2x+1)\,dx \;\;;$$

Use a constant that allows the curve to shift up and down, as the example does.



Substitution 201:

What is:

$$\int f(g(x))\, g'(x)\, dx = \int f(u) du$$

Substitution 201:

We assert that:

$$\int f(g(x))\, g'(x)\, dx = \int f(u) du$$

but what is:

$$\int f(g(x))\, g'(x)\, dx = \ ?\ ;$$

Use substitution:

Let $u = g(x)$

Then $du = d(g(x))dx$

Because

$$\frac{d}{dx}u = \frac{d}{dx}g(x) = g'(x)$$

AND IF $\dfrac{du}{dx} = g'(x)$

THEN

$$du = g'(x)\, dx$$

Substituting we have:

$$\int f(u)du = \int f(g(x))g'(x)dx$$

Substitution 301:

An advanced example of:

$$\int f(g(x))\, g'(x)\, dx = \int f(u) du$$

Substitution 301:

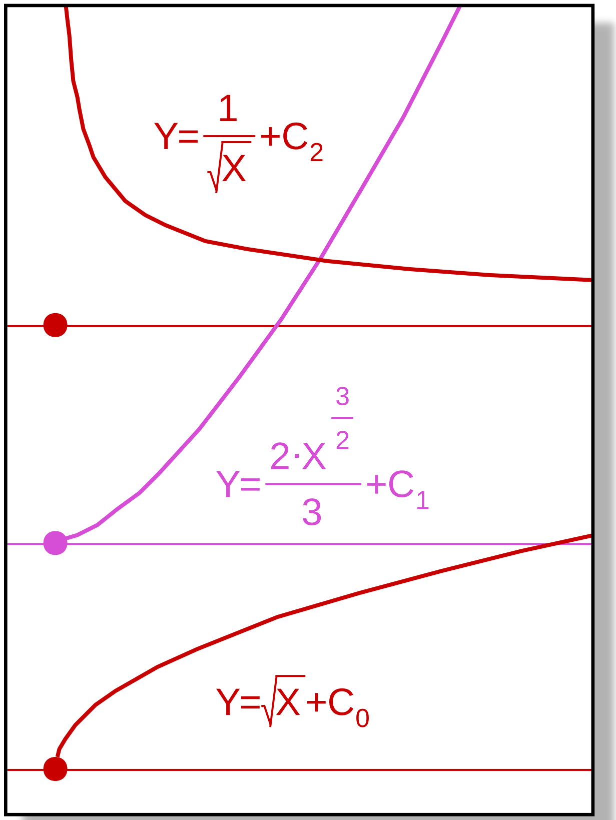
Lecture24-Substitution301.gx

We know that:

$$\int \sqrt{x}\, dx = \frac{x^{3/2}}{3/2} = \frac{2}{3} x^{3/2}$$

but what is:

$$\int \sqrt{\frac{1}{x}}\, dx = \; ?\; ;$$



Substitution 301:

Given $\int \sqrt{x^{-1}}\,dx$

If $u = x^{-1}$ *Then* $du = -x^{-2}dx$

We recognize that therefore $du = -u^{2}dx$

and by rearranging obtain $dx = -u^{-2}du$

So that $\int \sqrt{x^{-1}}\,dx = -\int \sqrt{u}\;u^{-2}du$

Simplifying $-\int \sqrt{u}\;u^{-2}du = -\int u^{-3/2}\,du = 2u^{-1/2} + C$

Finally we obtain : $2u^{-1/2} + C = 2(x^{-1})^{-1/2} + C = 2\sqrt{x}$

Substitution 301:

Lecture24-Substitution301.wxm

```
integrate(x^(1/2),x);
```

$$\frac{2\,x^{3/2}}{3}$$

```
integrate((1/x)^(1/2),x);
```

$$2\,\sqrt{x}$$

```
-sqrt(u)*u^(-2);
```

$$-\frac{1}{u^{3/2}}$$

```
integrate(%,u);
```

$$\frac{2}{\sqrt{u}}$$

```
ratsubst(1/x, u, %);
```

$$2\,\sqrt{x}$$

We can also use wxMaxima™ to assist in the substitution and check our work. Doing complex math by hand is both tedious and prone to error.

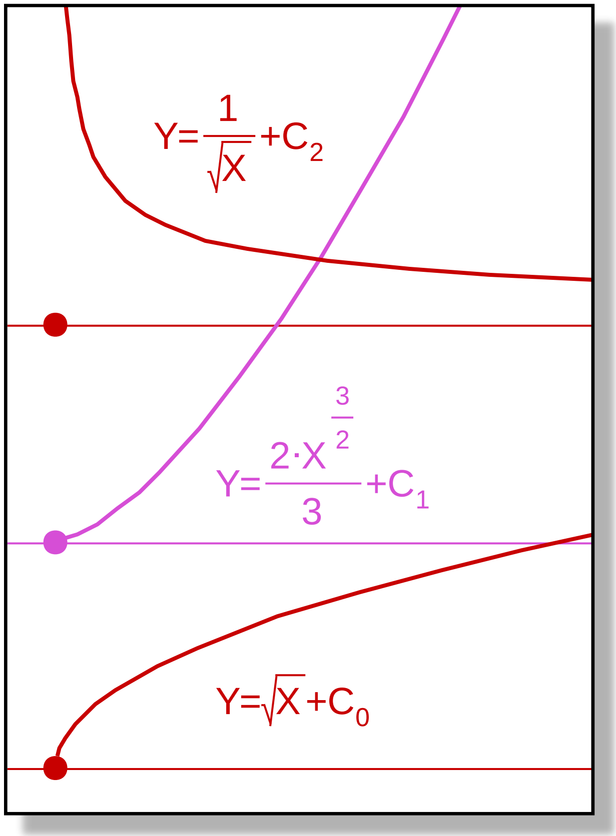
Substitution 301:

Lecture24-Substitution301.gx

We could have just noticed that:

$$\int \sqrt{\frac{1}{x}}dx = \int x^{-1/2}dx = 2\sqrt{x}$$

and avoided using substitution,but substitution is a useful technique so it pays to practice!



Substitution 301:

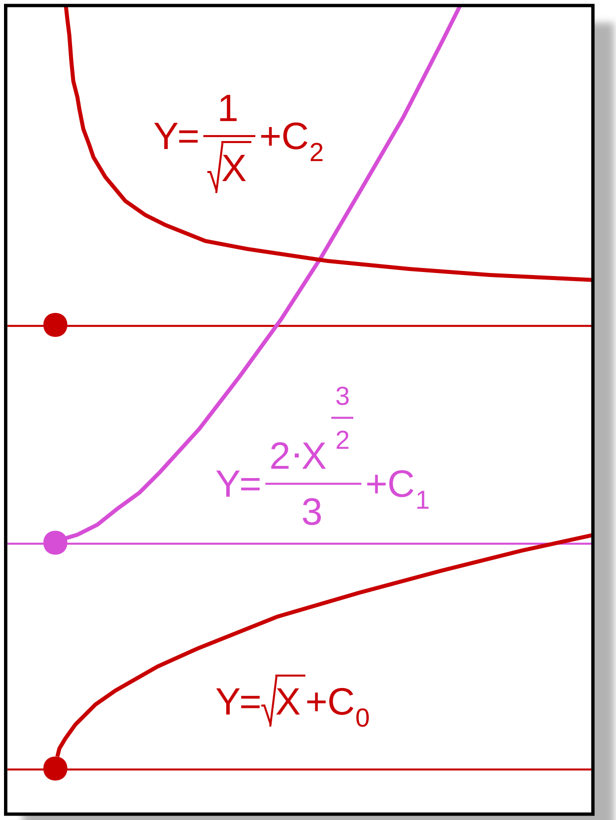
Lecture24-Substitution301.gx

Exercise:

Fill in the missing fourth curve which solves:

$$\int \sqrt{\frac{1}{x}}\, dx \;;$$

As before, use a constant that allows the curve to shift up and down.



```
The quick brown fox jumps over the lazy dog.
```

```
substitute(“dog”, “cat”)
```

```
The quick brown fox jumps over the lazy cat.
```

```
substitute(“fox”, “mouse”)
```

```
The quick brown mouse jumps over the lazy cat.
```

End